Keystone Exams: Algebra II

Assessment Anchors and Eligible Content

with Sample Questions and Glossary

Pennsylvania Department of Education

www.education.state.pa.us

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PENNSYLVANIA DEPARTMENT OF EDUCATION

General Introduction to the Keystone Exam Assessment Anchors

Introduction

Since the introduction of the Keystone Exams, the Pennsylvania Department of Education (PDE) has been working to create a set of tools designed to help educators improve instructional practices and better understand the Keystone Exams. The Assessment Anchors, as defined by the Eligible Content, are one of the many tools the Department believes will better align curriculum, instruction, and assessment practices throughout the Commonwealth. Without this alignment, it will not be possible to significantly improve student achievement across the Commonwealth.

How were Keystone Exam Assessment Anchors developed?

Prior to the development of the Assessment Anchors, multiple groups of PA educators convened to create a set of standards for each of the Keystone Exams. Enhanced Standards, derived from a review of existing standards, focused on what students need to know and be able to do in order to be college and career ready. (Note: Since that time, PA Core Standards have replaced the Enhanced Standards and reflect the college- and career-ready focus.) Additionally, the Assessment Anchors and Eligible Content statements were created by other groups of educators charged with the task of clarifying the standards assessed on the Keystone Exams. The Assessment Anchors, as defined by the Eligible Content, have been designed to hold together, or *anchor*, the state assessment system and the curriculum/instructional practices in schools.

Assessment Anchors, as defined by the Eligible Content, were created with the following design parameters:

- **Clear:** The Assessment Anchors are easy to read and are user friendly; they clearly detail which standards are assessed on the Keystone Exams.
- **Focused:** The Assessment Anchors identify a core set of standards that can be reasonably assessed on a large-scale assessment; this will keep educators from having to guess which standards are critical.
- **Rigorous:** The Assessment Anchors support the rigor of the state standards by assessing higher-order and reasoning skills.
- **Manageable:** The Assessment Anchors define the standards in a way that can be easily incorporated into a course to prepare students for success.

How can teachers, administrators, schools, and districts use these Assessment Anchors?

The Assessment Anchors, as defined by the Eligible Content, can help focus teaching and learning because they are clear, manageable, and closely aligned with the Keystone Exams. Teachers and administrators will be better informed about which standards will be assessed. The Assessment Anchors and Eligible Content should be used along with the Standards and the Curriculum Framework of the Standards Aligned System (SAS) to build curriculum, design lessons, and support student achievement.

The Assessment Anchors and Eligible Content are designed to enable educators to determine when they feel students are prepared to be successful in the Keystone Exams. An evaluation of current course offerings, through the lens of what is assessed on those particular Keystone Exams, may provide an opportunity for an alignment to ensure student preparedness.

How are the Assessment Anchors organized?

The Assessment Anchors, as defined by the Eligible Content, are organized into cohesive blueprints, each structured with a common labeling system that can be read like an outline. This framework is organized first by module, then by Assessment Anchor, followed by Anchor Descriptor, and then finally, at the greatest level of detail, by an Eligible Content statement. The common format of this outline is followed across the Keystone Exams.

Here is a description of each level in the labeling system for the Keystone Exams:

- **Module:** The Assessment Anchors are organized into two thematic modules for each of the Keystone Exams. The module title appears at the top of each page. The module level is important because the Keystone Exams are built using a module format, with each of the Keystone Exams divided into two equal-size test modules. Each module is made up of two or more Assessment Anchors.
- **Assessment Anchor:** The Assessment Anchor appears in the shaded bar across the top of each Assessment Anchor table. The Assessment Anchors represent categories of subject matter that anchor the content of the Keystone Exams. Each Assessment Anchor is part of a module and has one or more Anchor Descriptors unified under it.
- **Anchor Descriptor:** Below each Assessment Anchor is a specific Anchor Descriptor. The Anchor Descriptor level provides further details that delineate the scope of content covered by the Assessment Anchor. Each Anchor Descriptor is part of an Assessment Anchor and has one or more Eligible Content statements unified under it.
- **Eligible Content:** The column to the right of the Anchor Descriptor contains the Eligible Content statements. The Eligible Content is the most specific description of the content that is assessed on the Keystone Exams. This level is considered the assessment limit and helps educators identify the range of the content covered on the Keystone Exams.
- **PA Core Standard:** In the column to the right of each Eligible Content statement is a code representing one or more PA Core Standards that correlate to the Eligible Content statement. Some Eligible Content statements include annotations that indicate certain clarifications about the scope of an Eligible Content.
    - "e.g." ("for example")—sample approach, but not a limit to the Eligible Content
    - "Note"—content exclusions or definable range of the Eligible Content

How do the K–12 Pennsylvania Core Standards affect this document?

Assessment Anchor and Eligible Content statements are aligned to the PA Core Standards; thus, the former enhanced standards are no longer necessary. Within this document, all standard references reflect the PA Core Standards.

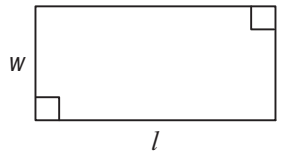
Standards Aligned System—www.pdesas.org

Pennsylvania Department of Education—www.education.state.pa.us

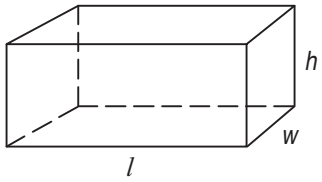
FORMULA SHEET

Formulas that you may need to work questions in this document are found below.
You may use calculator π or the number 3.14.

Shapes



$A = lw$



$V = lwh$

Data Analysis

Permutation: $_nP_r = \dfrac{n!}{(n-r)!}$

Combination: $_nC_r = \dfrac{n!}{r!(n-r)!}$

Exponential Properties

$a^m \cdot a^n = a^{m+n}$ $\qquad$ $(a^m)^n = a^{m \cdot n}$

$\dfrac{a^m}{a^n} = a^{m-n}$ $\qquad$ $a^{-1} = \dfrac{1}{a}$

Powers of the Imaginary Unit

$i = \sqrt{-1}$ $\qquad$ $i^2 = -1$

$i^3 = -i$ $\qquad$ $i^4 = 1$

Logarithmic Properties

$\log_a x = y \leftrightarrow x = a^y$ $\qquad$ $\log x = y \leftrightarrow x = 10^y$ $\qquad$ $\ln x = y \leftrightarrow x = e^y$

$\log_a(x \cdot y) = \log_a x + \log_a y$

$\log_a x^p = p \cdot \log_a x$

$\log_a \dfrac{x}{y} = \log_a x - \log_a y$

Quadratic Functions

General Formula: $f(x) = ax^2 + bx + c$

Standard (Vertex) Form: $f(x) = a(x - h)^2 + k$

Factored Form: $f(x) = a(x - x_1)(x - x_2)$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

when $ax^2 + bx + c = 0$ and $a \neq 0$

Compound Interest Equations

Annual: $A = P(1 + r)^t$

Periodic: $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

Continuous: $A = Pe^{rt}$

A = account total after t years
P = principal amount
r = annual rate of interest
t = time (years)
n = number of periods interest is compounded per year

ASSESSMENT ANCHOR A2.1.1 Operations with Complex Numbers				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
A2.1.1.1	Represent and/or use imaginary numbers in equivalent forms (e.g., square roots and exponents).	**A2.1.1.1.1**	Simplify/write square roots in terms of i (e.g., $\sqrt{-24} = 2i\sqrt{6}$).	**CC.2.1.HS.F.6**
		A2.1.1.1.2	Simplify/evaluate expressions involving powers of i (e.g., $i^6 + i^3 = -1 - i$).	

Sample Exam Questions

Standard A2.1.1.1.1

The expression $\sqrt{x}$ is equivalent to $14i\sqrt{3}$. What is the value of x?

A. -588

B. $-588i$

C. 588

D. $588i$

Standard A2.1.1.1.2

An expression is shown below.

$$x^5 + 6x^3 + 8x$$

Which value of x makes the expression equal to 0?

A. $-2i^2$

B. $-2i$

C. $4i$

D. $4i^2$

ASSESSMENT ANCHOR A2.1.1 Operations with Complex Numbers				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
A2.1.1.2	Apply the order of operations in computation and in problem-solving situations.	**A2.1.1.2.1**	Add and subtract complex numbers (e.g., $(7 - 3i) - (2 + i) = 5 - 4i$).	**CC.2.1.HS.F.6**
		A2.1.1.2.2	Multiply and divide complex numbers (e.g., $(7 - 3i)(2 + i) = 17 + i$).	

Sample Exam Questions

Standard A2.1.1.2.1

An equation with real numbers a, b, c, and d is shown below.

$$(4i + ab) - (6i + cd) = -2i$$

Which relationship **must** be true?

A. $ab = -cd$

B. $ab = cd$

C. $a = c$

D. $(a - c) = (b - d)$

Standard A2.1.1.2.2

An equation is shown below.

$$(a + bi)(4 - 2i) = 40$$

What is the value of b?

A. 2

B. 4

C. 10

D. 20

ASSESSMENT ANCHOR
A2.1.1 Operations with Complex Numbers

Sample Exam Questions

Standard A2.1.1

Lily is practicing multiplying complex numbers using the complex number $(2 + i)$.

To determine the value of $(2 + i)^2$, Lily performs the following operations:

step 1: $(2 + i)^2 = 4 + i^2$

step 2: $4 + i^2 = 4 + (-1)$

step 3: $4 + (-1) = 3$

Lily made an error.

A. Explain Lily's error and correct the step which contains her error.

Lily says that $(2 + i)^n$ is a complex number for every positive integer value of n.

B. Explain how you know that Lily is correct.

Continued on next page.

Continued. Please refer to the previous page for task explanation.

Lily is continuing to explore different ways in which complex numbers can be multiplied so the answer is **not** a complex number. Lily multiplies $(2 + i)$ and $(a + bi)$, where a and b are real numbers, and finds that her answer is **not** a complex number.

C. Write an equation that expresses the relationship between a and b.

equation: _____

D. Explain why the expression $(c + di)^2$ is **always** a complex number for nonzero, real values of c and d.

Standard A2.1.1

To find the roots of a quadratic equation, $ax^2 + bx + c$, where a, b, and c are real numbers, Jan uses the quadratic formula.

Jan finds that a quadratic equation has 2 distinct roots, but neither are real numbers.

A. Write an inequality using the variables a, b, and c that **must always** be true for Jan's quadratic equation.

inequality: _____

The expression $3 + \sqrt{-4}$ is a solution of the quadratic equation $x^2 - 6x + 13 = 0$.

B. What is $3 + \sqrt{-4}$ written as a complex number?

$3 + \sqrt{-4}$ = _____

Continued on next page.

Continued. Please refer to the previous page for task explanation.

C. What is $(5 + 2i)^2$ expressed as a complex number? Use the form $a + bi$, where a and b are real numbers.

$(5 + 2i)^2 =$ _____

D. What is a possible solution to the equation $5 = \sqrt{(a - bi)(a + bi)}$ when a and b are whole numbers greater than zero?

$a =$ _____ $b =$ _____

ASSESSMENT ANCHOR
A2.1.2 Non-Linear Expressions

Anchor Descriptor		Eligible Content		PA Core Standards
A2.1.2.1	Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.	**A2.1.2.1.1**	Use exponential expressions to represent rational numbers.	**CC.2.1.HS.F.1** **CC.2.2.HS.D.2**
		A2.1.2.1.2	Simplify/evaluate expressions involving positive and negative exponents and/or roots (may contain all types of real numbers—exponents should not exceed power of 10).	
		A2.1.2.1.3	Simplify/evaluate expressions involving multiplying with exponents (e.g., $x^6 \bullet x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7 = x^{42}$), and powers of products (e.g., $(2x^2)^3 = 8x^6$). Note: Limit to rational exponents.	
		A2.1.2.1.4	Simplify or evaluate expressions involving logarithms and exponents (e.g., $\log_2 8 = 3$ or $\log_4 2 = \frac{1}{2}$).	

Sample Exam Questions

Standard A2.1.2.1.1

Which expression is equivalent to $\frac{1}{25}$?

A. $4 \bullet 10^{-2}$

B. $25 \bullet 10^{-2}$

C. $4 \bullet 10^{-1}$

D. $25 \bullet 10^{-1}$

Standard A2.1.2.1.2

An expression is shown below.

$$\sqrt[8]{x^{-8x}} + \sqrt[5]{x^{-5x}}$$

Which value of x makes the expression equal to $\frac{1}{2}$?

A. 0

B. 1

C. 2

D. 4

MODULE 1—Number Systems and Non-Linear Expressions & Equations

Standard A2.1.2.1.3

An equation is shown below.

$$x^{10n}(x^5 \bullet (x^{-5})^n) = x^{-10}$$

Which is the value of n?

A. -3

B. -1

C. $-\frac{1}{2}$

D. 2

Standard A2.1.2.1.4

An expression is shown below.

$$\log \sqrt[4]{\frac{x^{16}}{y^4}}$$

What is the value of the expression when $\log x = 8$ and $\log y = 1$?

A. 7

B. 15

C. 16

D. 31

ASSESSMENT ANCHOR A2.1.2 Non-Linear Expressions				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
A2.1.2.2	Simplify expressions involving polynomials.	**A2.1.2.2.1**	Factor algebraic expressions, including difference of squares and trinomials. Note: Trinomials limited to the form ax^2+bx+c where a is not equal to 0.	**CC.2.2.HS.D.1** **CC.2.2.HS.D.2** **CC.2.2.HS.D.3** **CC.2.2.HS.D.4** **CC.2.2.HS.D.5**
		A2.1.2.2.2	Simplify rational algebraic expressions.	

Sample Exam Questions

Standard A2.1.2.2.1

An expression is shown below.

$$6x^2 - 19x + 10$$

Which is a factor of the expression?

A. $2x + 2$

B. $2x + 5$

C. $3x - 2$

D. $3x - 5$

Standard A2.1.2.2.2

An expression is shown below.

$$\frac{3x^2 - 4x - 15}{2x^2 - 5x - 3}; \quad x \neq -\frac{1}{2}, 3$$

Which expression is equivalent to the one shown?

A. $\frac{3x + 5}{2x + 1}$

B. $\frac{3x - 5}{2x - 1}$

C. $x^2 + x - 12$

D. $5x^2 - 9x - 18$

ASSESSMENT ANCHOR
A2.1.2 Non-Linear Expressions

Sample Exam Questions

Standard A2.1.2

The expression $(10^d)^{3/2}$ is used to find how many times more energy is released by an earthquake of greater magnitude than by an earthquake of lesser magnitude, where d is the **difference** in magnitudes.

A. How many times more energy is given off by an earthquake with magnitude 5.2 than by an earthquake with magnitude 3.2?

times more energy: _____

B. What is the difference (d) when 100 times more energy is released by an earthquake of greater magnitude than by an earthquake of lesser magnitude?

d = _____

Continued on next page.

Continued. Please refer to the previous page for task explanation.

C. What is an equivalent exponential expression to $(10^d)^{3/2}$ with a base of 1,000?

equivalent exponential expression: _____

D. Explain why $(\sqrt{10})^{3d}$ is equivalent to $(10^d)^{3/2}$.

Standard A2.1.2

Beatriz is simplifying exponential and radical expressions.

A. What rational number is the result of simplifying $\sqrt[3]{16^{-3/2}}$?

rational number: _____

The exponential expression 5^{2x+3} can be simplified to the form $a(b^x)$ where a and b are integers.

B. What are the values of a and b?

a = _____

b = _____

Continued on next page.

Continued. Please refer to the previous page for task explanation.

The variable c represents a whole number between 1 and 100. The values of the expressions $c^{1/2}$ and $c^{2/3}$ are both whole numbers for only one value of c.

C. What whole number does c represent?

c = _____

ASSESSMENT ANCHOR A2.1.3 Non-Linear Equations				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
A2.1.3.1	Write and/or solve non-linear equations using various methods.	**A2.1.3.1.1**	Write and/or solve quadratic equations (including factoring and using the Quadratic Formula).	**CC.2.2.HS.C.2** **CC.2.2.HS.C.3** **CC.2.2.HS.C.4** **CC.2.2.HS.C.5** **CC.2.2.HS.C.6** **CC.2.2.HS.D.5** **CC.2.2.HS.D.6** **CC.2.2.HS.D.7** **CC.2.2.HS.D.8** **CC.2.2.HS.D.9** **CC.2.2.HS.D.10**
		A2.1.3.1.2	Solve equations involving rational and/or radical expressions (e.g., $10/(x + 3) + 12/(x - 2) = 1$ or $\sqrt{x^2 + 21x} = 14$).	
		A2.1.3.1.3	Write and/or solve a simple exponential or logarithmic equation (including common and natural logarithms).	
		A2.1.3.1.4	Write, solve, and/or apply linear or exponential growth or decay (including problem situations).	

Sample Exam Questions

Standard A2.1.3.1.1

The equation $x^2 + bx + c = 0$ has **exactly** 1 real solution when b and c are real numbers. Which equation describes b in terms of c?

A. $b = c^2$

B. $b = \sqrt{c}$

C. $b = 2c$

D. $b = 2\sqrt{c}$

Standard A2.1.3.1.2

An equation is shown below.

$$\frac{64x^4 + 8x^3}{8x^3} = x^2 + 8$$

What is the solution set of the equation?

A. $\{-7, -1\}$

B. $\{-7, 1\}$

C. $\{-1, 7\}$

D. $\{1, 7\}$

MODULE 1—Number Systems and Non-Linear Expressions & Equations

Standard A2.1.3.1.3

An equation is shown below.

$$3^{5x} = 9^{2x-1}$$

Which equation has the same solution?

A. $3x = 10x - 5$

B. $5x = 4x - 2$

C. $8x = 11x - 1$

D. $15x = 18x - 9$

Standard A2.1.3.1.4

A patient is given a 100-milligram dosage of a drug that decays exponentially, with a half-life of 6 hours. Which equation could be used to find the milligrams of drug remaining (y) after x hours?

A. $y = 100(6)^{0.5x}$

B. $y = 100(x)^{0.5/6}$

C. $y = 100(0.5)^{x/6}$

D. $y = 100(0.5x)^{1/6}$

ASSESSMENT ANCHOR A2.1.3 Non-Linear Equations				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
A2.1.3.2	Describe and/or determine change.	**A2.1.3.2.1**	Determine how a change in one variable relates to a change in a second variable (e.g., $y = 4/x$; if x doubles, what happens to y?).	**CC.2.2.HS.C.2** **CC.2.2.HS.C.3** **CC.2.2.HS.C.4** **CC.2.2.HS.D.7** **CC.2.2.HS.D.8** **CC.2.2.HS.D.9**
		A2.1.3.2.2	Use algebraic processes to solve a formula for a given variable (e.g., solve $d = rt$ for r).	

Sample Exam Questions

Standard A2.1.3.2.1

A moving object's kinetic energy (E_k) is dependent on the mass of the object (m) and the object's velocity (v), as shown in the equation below.

$$E_k = \frac{1}{2}mv^2$$

How does the value of E_k change when the value of m is unchanged and the value of v is multiplied by 2?

A. The value of E_k is squared.

B. The value of E_k is multiplied by 2.

C. The value of E_k is multiplied by 4.

D. The value of E_k is multiplied by 8.

Standard A2.1.3.2.2

Physicists use the formula shown below to determine total energy (E) of a body by using momentum (p), mass (m), and the speed of light (c).

$$E = \sqrt{(pc)^2 + (mc^2)^2}$$

A physicist knows the speed of light, the mass of the body, and the total energy used by the body. Which formula could be used to determine the momentum of the body?

A. $p = \frac{\sqrt{E^2 - m^2c^4}}{c}$

B. $p = \frac{\sqrt{E - mc^2}}{c}$

C. $p = \frac{E}{c} - mc$

D. $p = E - mc$

ASSESSMENT ANCHOR
A2.1.3 Non-Linear Equations

Sample Exam Questions

Standard A2.1.3

Michaela is solving rational equations.

A. What is the solution set of the equation $\frac{x^2 - 7x + 12}{x^2 + x - 12} = 3$? Show or explain all your work.

B. The only solution of the equation $\frac{x^2 + bx - 18}{-2x + 4} = 4$ is $x = -17$. What is the value of b?

$b =$ _____

Continued on next page.

Continued. Please refer to the previous page for task explanation.

Michaela solved for x in the rational equation as shown below.

$$\frac{x^2 + 2x - 15}{x - 3} = 3$$

$$(x - 3) \bullet \frac{x^2 + 2x - 15}{x - 3} = 3 \bullet (x - 3)$$

$$x^2 + 2x - 15 = 3x - 9$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x - 3 = 0 \text{ or } x + 2 = 0$$

$$x = \{3, -2\}$$

The solution set for x is incorrect.

C. Explain Michaela's error.

Standard A2.1.3

A fully-charged lithium-ion computer battery loses 20% of its permanent capacity each year of storage.

A. Write an exponential equation showing the capacity (c) remaining in a fully-charged lithium-ion computer battery after y years.

$c =$ _____

B. What capacity is remaining in a fully-charged lithium-ion battery after 1.5 years?

capacity: _____

Continued on next page.

Continued. Please refer to the previous page for task explanation.

C. Solve the exponential equation from **part A** for y.

$y =$ _____

D. For how many years will a fully-charged lithium-ion computer battery have been stored when it has lost exactly half of its capacity?

years: _____

MODULE 2—Functions and Data Analysis

ASSESSMENT ANCHOR
A2.2.1 Patterns, Relations, and Functions

Anchor Descriptor		Eligible Content		PA Core Standards
A2.2.1.1	Analyze and/or use patterns or numbers.	**A2.2.1.1.1**	Analyze a set of data for the existence of a pattern, and represent the pattern with a rule algebraically and/or graphically.	**CC.2.1.HS.F.7** **CC.2.2.HS.C.1** **CC.2.2.HS.C.2** **CC.2.2.HS.C.3** **CC.2.2.HS.C.5** **CC.2.2.HS.C.6** **CC.2.3.HS.A.10** **CC.2.4.HS.B.2**
		A2.2.1.1.2	Identify and/or extend a pattern as either an arithmetic or geometric sequence (e.g., given a geometric sequence, find the 20th term).	
		A2.2.1.1.3	Determine the domain, range, or inverse of a relation.	
		A2.2.1.1.4	Identify and/or determine the characteristics of an exponential, quadratic, or polynomial function (e.g., intervals of increase/decrease, intercepts, zeros, and asymptotes).	

Sample Exam Questions

Standard A2.2.1.1.1

Terms 1 through 5 of a pattern are listed below.

4 3 4 7 12

The pattern continues. Which expression could be used to determine the *n*th term in the pattern?

A. $2n + 2$

B. $|n - 2| + 3$

C. $n^2 - 4n + 7$

D. $n^3 - 5n^2 + 7n + 1$

Standard A2.2.1.1.2

Terms 1 through 5 of a sequence are shown below.

$\frac{8}{81}$ $\frac{4}{27}$ $\frac{2}{9}$ $\frac{1}{3}$ $\frac{1}{2}$

What is the 10th term in the sequence?

A. $\frac{81}{64}$

B. $\frac{81}{32}$

C. $\frac{243}{64}$

D. $\frac{243}{32}$

MODULE 2—Functions and Data Analysis

Standard A2.2.1.1.3

What is the inverse of $y = \ln(x - 15) + 3$?

A. $y = e^{x-3} + 15$

B. $y = e^{x} + 12$

C. $y = e^{x+12} + 15$

D. $y = e^{x-15} + 3$

Standard A2.2.1.1.4

When is $f(x) = x^2 - x - 12$ increasing?

A. $x > \frac{1}{2}$

B. $x < \frac{1}{2}$

C. $x > -3$

D. $x < 4$

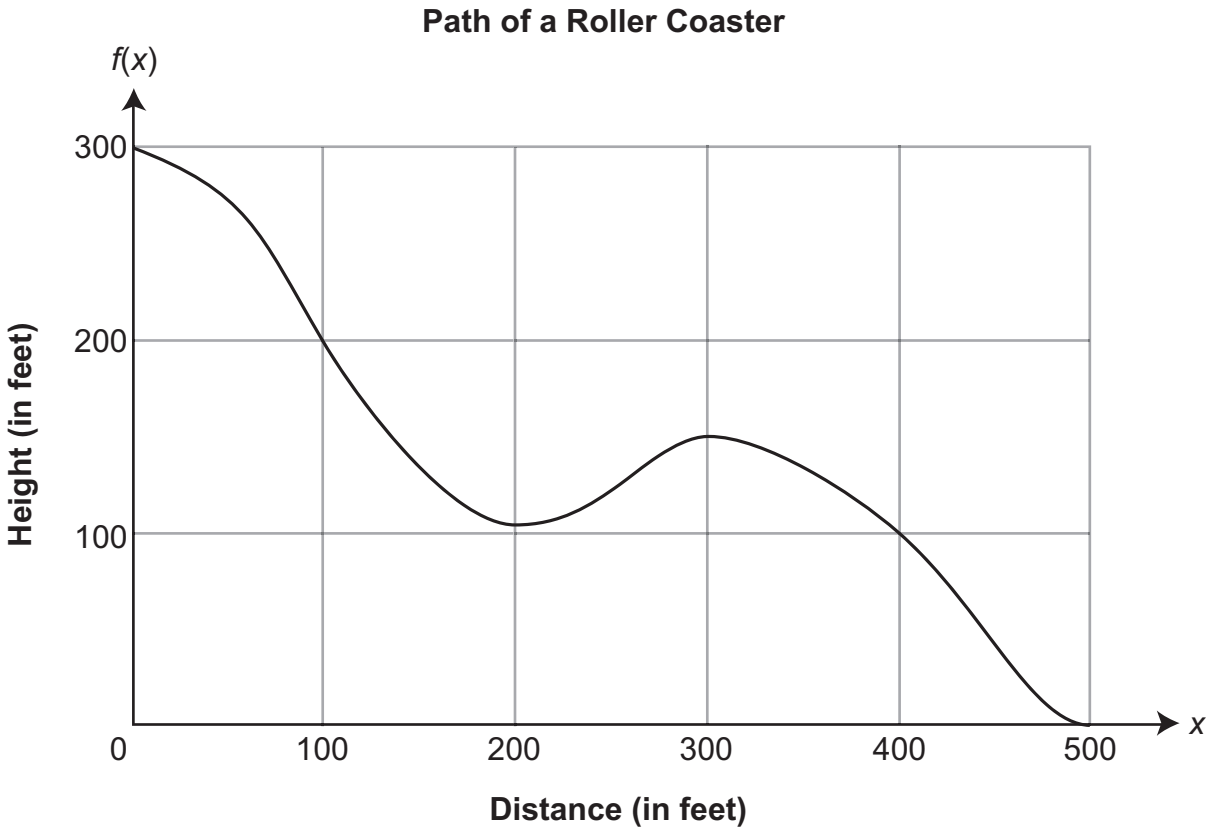
MODULE 2—Functions and Data Analysis

ASSESSMENT ANCHOR
A2.2.1 Patterns, Relations, and Functions

Sample Exam Questions

Standard A2.2.1

The path of a roller coaster after it has reached the top of the first hill follows a polynomial function, as shown in the graph below.



A. Over what interval is $f(x)$ increasing?

_____ ≤ x ≤ _____

Continued on next page.

MODULE 2—Functions and Data Analysis

Continued. Please refer to the previous page for task explanation.

B. At what value of x is there a minimum of $f(x)$ over the interval $0 \leq x \leq 300$?

value of x: _____

C. At what value of x is there a zero of $f(x)$?

value of x: _____

Continued on next page.

MODULE 2—Functions and Data Analysis

Continued. Please refer to the previous page for task explanation.

D. Explain why $f(x)$ **cannot** be a quadratic function.

MODULE 2—Functions and Data Analysis

Standard A2.2.1

The function below describes the graph of a quadratic function where c is a positive real number.

$$y = x^2 - c$$

A. What are the x-intercept(s) of the graph of the quadratic function?

x-intercept(s): _____

B. What is a y-value, in terms of c, which **cannot** be in the range of the quadratic function?

y-value: _____

Continued on next page.

MODULE 2—Functions and Data Analysis

Continued. Please refer to the previous page for task explanation.

C. What is the domain of the inverse of the quadratic function?

domain of the inverse of the quadratic function: _____

MODULE 2—Functions and Data Analysis

ASSESSMENT ANCHOR
A2.2.2 Applications of Functions

Anchor Descriptor		Eligible Content		PA Core Standards
A2.2.2.1	Create, interpret, and/or use polynomial, exponential, and/or logarithmic functions and their equations, graphs, or tables.	**A2.2.2.1.1**	Create, interpret, and/or use the equation, graph, or table of a polynomial function (including quadratics).	**CC.2.1.HS.F.3** **CC.2.1.HS.F.4** **CC.2.2.HS.C.3** **CC.2.2.HS.C.4** **CC.2.2.HS.C.5** **CC.2.2.HS.C.6** **CC.2.2.HS.D.7** **CC.2.3.HS.A.10**
		A2.2.2.1.2	Create, interpret, and/or use the equation, graph, or table of an exponential or logarithmic function (including common and natural logarithms).	
		A2.2.2.1.3	Determine, use, and/or interpret minimum and maximum values over a specified interval of a graph of a polynomial, exponential, or logarithmic function.	
		A2.2.2.1.4	Translate a polynomial, exponential, or logarithmic function from one representation of a function to another (graph, table, and equation).	

Sample Exam Questions

Standard A2.2.2.1.1

The table below represents a quadratic function.

x	$f(x)$
−5	−3
−4	0
−1	−3
0	−8
1	−15

Which describes a complete list where the zeros of $f(x)$ occur?

A. $x = 8$ and $x = 4$

B. $x = 4$ and $x = 2$

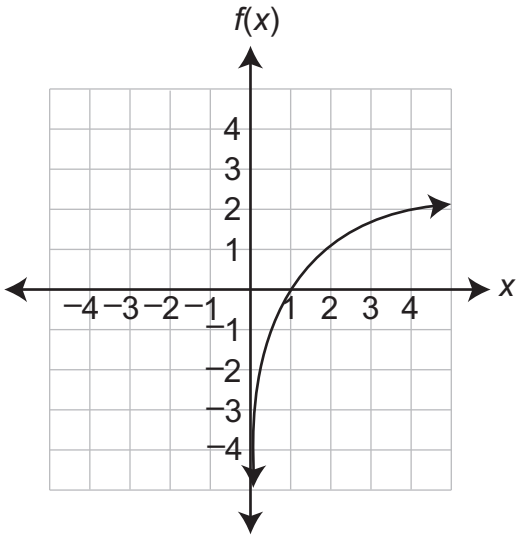
C. $x = -8$ and $x = -4$

D. $x = -4$ and $x = -2$

MODULE 2—Functions and Data Analysis

Standard A2.2.2.1.2

A logarithmic function is graphed below.



What is the value of $f(8)$?

A. 3

B. 4

C. 16

D. 256

Standard A2.2.2.1.3

A function of x is shown below.

$$y = -3(x - 2)(x + 4)$$

What is the maximum value of the function over the interval $-3 \leq x \leq 2$?

A. 0

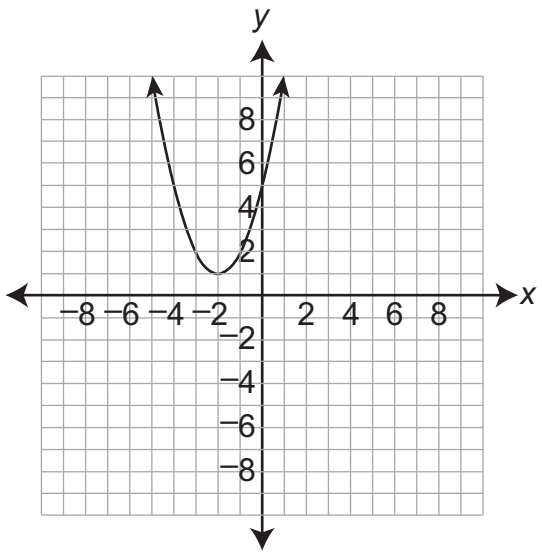
B. 15

C. 24

D. 27

Standard A2.2.2.1.4

A function of x is graphed below.



Which equation **best** describes the graph?

A. $y = x^2 + 5$

B. $y = (x - 2)^2 + 1$

C. $y = (x + 2)^2 + 1$

D. $y = (x + 2)(x - 1)$

MODULE 2—Functions and Data Analysis

ASSESSMENT ANCHOR A2.2.2 Applications of Functions				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
A2.2.2.2	Describe and/or determine families of functions.	**A2.2.2.2.1**	Identify or describe the effect of changing parameters within a family of functions (e.g., $y = x^2$ and $y = x^2 + 3$, or $y = x^2$ and $y = 3x^2$).	**CC.2.2.HS.C.4** **CC.2.2.HS.C.5** **CC.2.2.HS.C.6**

Sample Exam Question

Standard A2.2.2.2.1

The graph of the equation $y = 3x^2$ has its vertex at the coordinate point (0, 0). What coordinate point describes the vertex of the graph of the equation $y = 3x^2 - 3$?

A. (0, −3)

B. (0, 3)

C. (−3, 0)

D. (3, 0)

MODULE 2—Functions and Data Analysis

ASSESSMENT ANCHOR
A2.2.2 Applications of Functions

Sample Exam Questions

Standard A2.2.2

An exponential function of the form $f(x) = a \bullet b^x + c$ is represented by the pairs of values shown in the table below.

x	$f(x)$
−2	1,215
−1	135
0	15
1	$\frac{5}{3}$
2	$\frac{5}{27}$

A. Determine the exponential function that contains the 5 points shown in the table.

$f(x) =$ _____

Continued on next page.

MODULE 2—Functions and Data Analysis

Continued. Please refer to the previous page for task explanation.

B. What is the minimum value of $f(x)$ over the interval $-5 \leq x \leq 5$?

minimum: _____

C. Describe the difference in the graph of the exponential function $g(x) = a \bullet b^{x+2} + c$ and the graph of the exponential function $f(x) = a \bullet b^{x} + c$ when a, b, and c remain unchanged.

Continued on next page.

MODULE 2—Functions and Data Analysis

Continued. Please refer to the previous page for task explanation.

D. What is the value of $g(0)$?

$g(0) =$ _____

MODULE 2—Functions and Data Analysis

Standard A2.2.2

The number of meals a restaurant serves is a function of the price of each meal. The restaurant found it will serve 72 meals when it charges a price of $7.00 per meal. It will serve 52 meals when it charges a price of $12.00 per meal. The relationship between the number of meals served and the price of each meal is linear.

A. Write a linear function that represents the relationship between the number of meals served ($f(x)$) and the price of each meal (x).

$f(x) =$ _____

Continued on next page.

MODULE 2—Functions and Data Analysis

Continued. Please refer to the previous page for task explanation.

The table below shows profits for the same restaurant based on the price of each meal.

Restaurant Profits

Price (in dollars)	Profit (in dollars)
5	80
7	216
16	432
22	216

The relationship between the profit and the price of each meal can be represented by a quadratic function.

B. Write a quadratic function that represents the relationship between the restaurant's profit ($g(x)$) and the price of each meal (x).

$g(x) =$ _____

C. Based on the quadratic function from **part B**, what is a price per meal where the profit will be exactly $0.00?

price: $_____

D. Based on the quadratic function from **part B**, what is the maximum profit, in dollars?

maximum profit: $_____

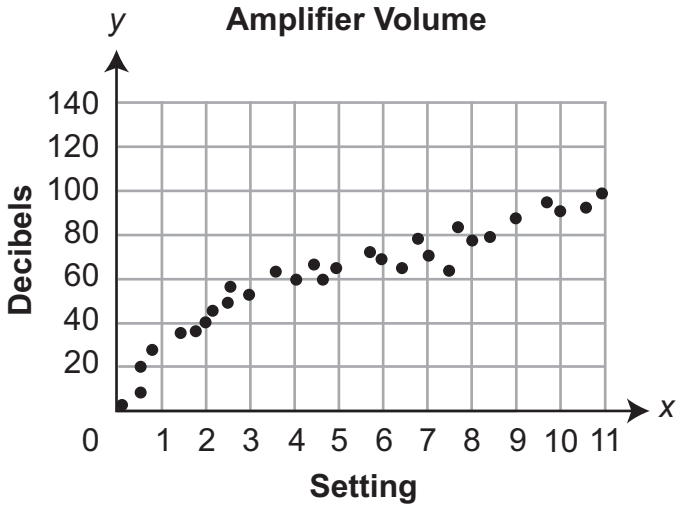
MODULE 2—Functions and Data Analysis

ASSESSMENT ANCHOR A2.2.3 Data Analysis				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
A2.2.3.1	Analyze and/or interpret data on a scatter plot and/or use a scatter plot to make predictions.	**A2.2.3.1.1**	Draw, identify, find, interpret, and/or write an equation for a regression model (lines and curves of best fit) for a scatter plot.	**CC.2.1.HS.F.3** **CC.2.1.HS.F.5** **CC.2.4.HS.B.2** **CC.2.4.HS.B.3**
		A2.2.3.1.2	Make predictions using the equations or graphs of regression models (lines and curves of best fit) of scatter plots.	

Sample Exam Questions

Standard A2.2.3.1.1

Nehla recorded the volume in decibels for different settings on her amplifier as shown in the scatter plot below.



Which equation best describes the curve of best fit?

A. $y = \frac{x}{9}$

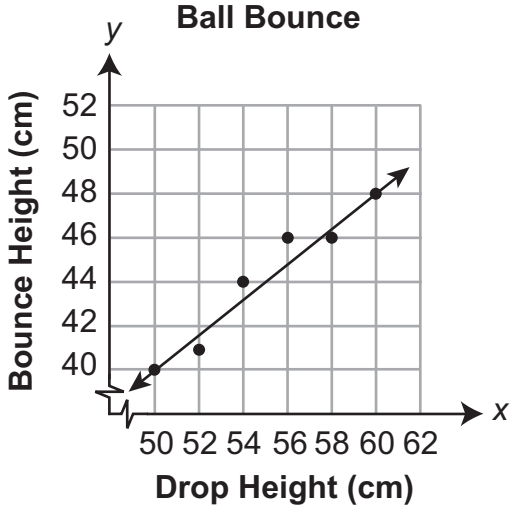
B. $y = 30x$

C. $y = 30x^2$

D. $y = 30\sqrt{x}$

Standard A2.2.3.1.2

Students conducted an experiment dropping balls and measuring how high the balls bounced. They recorded their results in the scatter plot shown below.



Based on the line of best fit shown on the scatter plot, which is most likely the height of the bounce of a ball that was dropped from a height of 30 centimeters (cm)?

A. 20 cm

B. 24 cm

C. 30 cm

D. 38 cm

MODULE 2—Functions and Data Analysis

ASSESSMENT ANCHOR A2.2.3 Data Analysis				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
A2.2.3.2	Apply probability to practical situations.	**A2.2.3.2.1**	Use combinations, permutations, and the fundamental counting principle to solve problems involving probability.	**CC.2.4.HS.B.4** **CC.2.4.HS.B.5** **CC.2.4.HS.B.6** **CC.2.4.HS.B.7**
		A2.2.3.2.2	Use odds to find probability and/or use probability to find odds.	
		A2.2.3.2.3	Use probability for independent, dependent, or compound events to predict outcomes.	

Sample Exam Questions

Standard A2.2.3.2.1

A 5-character key code is randomly generated by a computer using the 26 letters of the alphabet and the 10 digits 0–9. What is the probability that the 5 characters in a key code, listed as they are randomly generated, will spell the word "GREAT"?

A. $\frac{1}{60,466,176}$

B. $\frac{1}{45,239,040}$

C. $\frac{5}{60,466,176}$

D. $\frac{5}{45,239,040}$

Standard A2.2.3.2.2

The probability an airplane will be delayed is $\frac{5}{23}$.

What are the odds in favor of the airplane being delayed?

A. 5:23

B. 5:18

C. 18:23

D. 18:5

MODULE 2—Functions and Data Analysis

Standard A2.2.3.2.3

A candy store owner gives 2 sample jelly beans to each customer. The owner randomly selects the 2 samples from a large container of jelly beans. The list below shows the percent of each flavor of jelly bean in the container.

- 25% of the jelly beans are strawberry
- 40% of the jelly beans are green apple
- 20% of the jelly beans are grape
- 15% of the jelly beans are lemon

Last month 550 people each received 2 sample jelly beans. Based on the information above, which is **most likely** the number of people who received 1 lemon-flavored jelly bean and 1 green-apple-flavored jelly bean?

A. 10

B. 33

C. 66

D. 151

MODULE 2—Functions and Data Analysis

ASSESSMENT ANCHOR
A2.2.3 Data Analysis

Sample Exam Questions

Standard A2.2.3

A naturalist introduced a new species of fish to a lake. He started by putting 20 fish of the new species into the lake. The naturalist then recorded the total number of fish of the new species in the lake on the same day for each of the next three years. His data is listed below.

Fish of the New Species

Year	Number of Fish
0	20
1	96
2	158
3	204

The exponential curve of best fit for this data can be expressed in the form $y = -380\,(b)^x + c$, where x represents the year and y represents the number of fish. The base b and constant c are real numbers.

A. What is the equation of the exponential curve of best fit for the data?

equation: _____

Continued on next page.

MODULE 2—Functions and Data Analysis

Continued. Please refer to the previous page for task explanation.

B. Based on your equation of the curve of best fit from **part A**, how many fish of the new species will be in the lake when the naturalist records his data in year 6?

fish of the new species: _____

C. Based on your equation of the curve of best fit from **part A**, what is the first year in which there will be at least 350 fish of the new species in the lake?

year: _____

Continued on next page.

MODULE 2—Functions and Data Analysis

Continued. Please refer to the previous page for task explanation.

The naturalist wants to use the data to predict how many fish of the new species will be in the lake in 50 years.

D. What is one possible reason the correct equation for the curve of best fit may not provide an accurate prediction?

MODULE 2—Functions and Data Analysis

Standard A2.2.3

The cast of a play consists of 7 males and 5 females. There is 1 male lead role and 1 female lead role.

A. In how many different ways can the two lead roles be cast?

different ways: _____

Of the 7 males and 5 females, there are two sets of siblings. One set consists of a brother and sister, and the other set consists of a brother and his 2 sisters. The lead roles are cast randomly.

B. What is the probability that a pair of siblings are cast in the lead roles?

probability: _____

Continued on next page.

MODULE 2—Functions and Data Analysis

Continued. Please refer to the previous page for task explanation.

There are 8 speaking roles in the play. Of the 8 speaking roles, 5 are for males and 3 are for females. After the 8 speaking roles in the play are cast, the remaining cast members will make up the chorus.

C. What is the probability that the set of siblings consisting of a brother and his 2 sisters are all in the chorus?

probability: _____

The director decides to add *n* more females to the cast.

D. Write an expression that represents the probability that the set of siblings that consists of a brother and his 2 sisters are all in the chorus.

expression: _____

KEYSTONE ALGEBRA II ASSESSMENT ANCHORS
KEY TO SAMPLE MULTIPLE-CHOICE ITEMS

Algebra II

Eligible Content	Key
A2.1.1.1.1	A
A2.1.1.1.2	B
A2.1.1.2.1	B
A2.1.1.2.2	B

Eligible Content	Key
A2.1.2.1.1	A
A2.1.2.1.2	C
A2.1.2.1.3	A
A2.1.2.1.4	D
A2.1.2.2.1	C
A2.1.2.2.2	A

Eligible Content	Key
A2.1.3.1.1	D
A2.1.3.1.2	D
A2.1.3.1.3	B
A2.1.3.1.4	C
A2.1.3.2.1	C
A2.1.3.2.2	A

Eligible Content	Key
A2.2.1.1.1	C
A2.2.1.1.2	C
A2.2.1.1.3	A
A2.2.1.1.4	A

Eligible Content	Key
A2.2.2.1.1	D
A2.2.2.1.2	A
A2.2.2.1.3	D
A2.2.2.1.4	C
A2.2.2.2.1	A

Eligible Content	Key
A2.2.3.1.1	D
A2.2.3.1.2	B
A2.2.3.2.1	A
A2.2.3.2.2	B
A2.2.3.2.3	C

Keystone Exams: Algebra

Glossary to the Assessment Anchor & Eligible Content

The Keystone Glossary includes terms and definitions associated with the Keystone Assessment Anchors and Eligible Content. The terms and definitions included in the glossary are intended to assist Pennsylvania educators in better understanding the Keystone Assessment Anchors and Eligible Content. The glossary does not define all possible terms included on an actual Keystone Exam, and it is not intended to define terms for use in classroom instruction for a particular grade level or course.

Pennsylvania Department of Education

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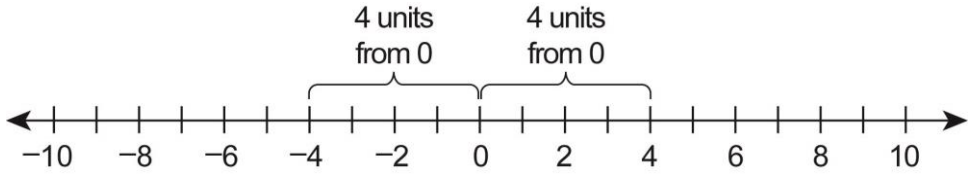
Absolute Value

A number's distance from zero on the number line. It is written $|a|$ and is read "the absolute value of a." It results in a number greater than or equal to zero (e.g., $|4| = 4$ and $|{}^{-}4| = 4$). Example of absolute values of $^{-}4$ and 4 on a number line:



Additive Inverse

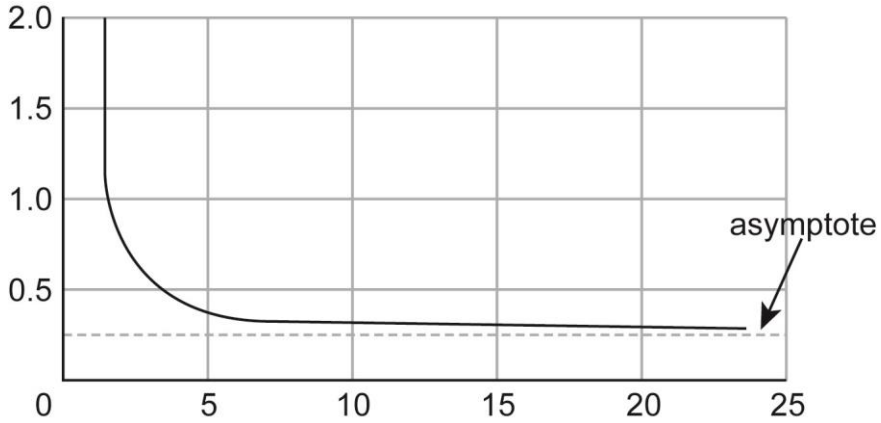
The opposite of a number (i.e., for any number a, the additive inverse is ^{-}a). Any number and its additive inverse will have a sum of zero (e.g., $^{-}4$ is the additive inverse of 4 since $4 + {}^{-}4 = 0$; likewise, the additive inverse of $^{-}4$ is 4 since $^{-}4 + 4 = 0$).

Arithmetic Sequence

An ordered list of numbers that increases or decreases at a constant rate (i.e., the difference between numbers remains the same). Example: 1, 7, 13, 19, … is an arithmetic sequence as it has a constant difference of +6 (i.e., 6 is added over and over).

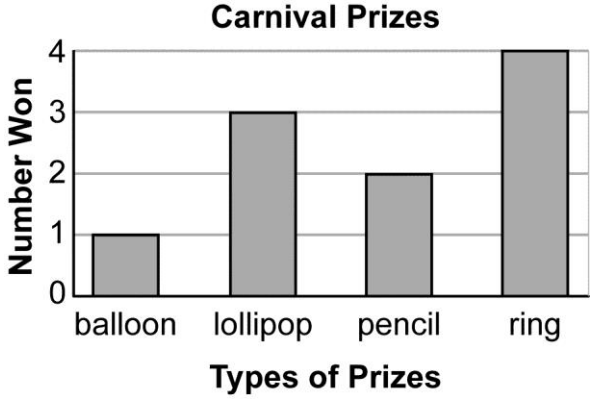
Asymptote

A straight line to which the curve of a graph comes closer and closer. The distance between the curve and the asymptote approaches zero as they tend to infinity. The asymptote is denoted by a dashed line on a graph. The most common asymptotes are horizontal and vertical. Example of a horizontal asymptote:



Bar Graph

A graph that shows a set of frequencies using bars of equal width, but heights that are proportional to the frequencies. It is used to summarize discrete data. Example of a bar graph:

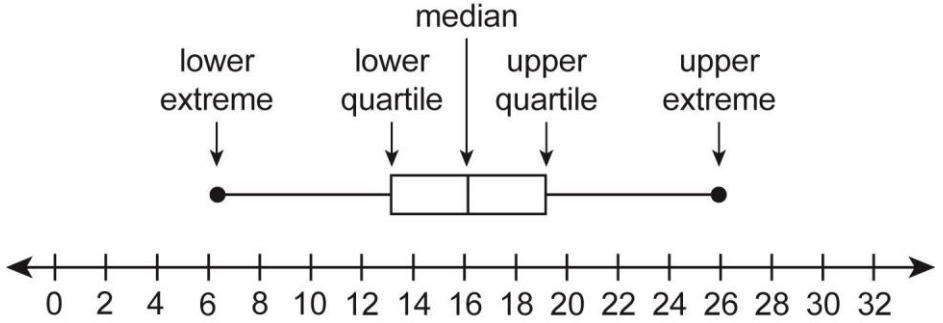


Binomial

A polynomial with two unlike terms (e.g., $3x + 4y$ or $a^3 - 4b^2$). Each term is a monomial, and the monomials are joined by an addition symbol (+) or a subtraction symbol (–). It is considered an algebraic expression.

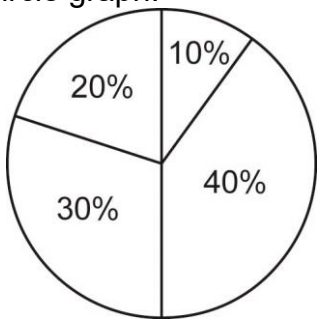
Box-and-Whisker Plot

A graphic method for showing a summary and distribution of data using median, quartiles, and extremes (i.e., minimum and maximum) of data. This shows how far apart and how evenly data is distributed. It is helpful when a visual is needed to see if a distribution is skewed or if there are any outliers. Example of a box-and-whisker plot:



Circle Graph (or Pie Chart)

A circular diagram using different-sized sectors of a circle whose angles at the center are proportional to the frequency. Sectors can be visually compared to show information (e.g., statistical data). Sectors resemble slices of a pie. Example of a circle graph:



Coefficient The number, usually a constant, that is multiplied by a variable in a term (e.g., 35 is the coefficient of $35x^2y$); the absence of a coefficient is the same as a 1 being present (e.g., x is the same as $1x$).

Combination An unordered arrangement, listing or selection of objects (e.g., two-letter combinations of the three letters X, Y, and Z would be XY, XZ, and YZ; XY is the same as YX and is not counted as a different combination). A combination is similar to, but not the same as, a permutation.

Common Logarithm A logarithm with base 10. It is written log x. The common logarithm is the power of 10 necessary to equal a given number (i.e., log $x = y$ is equivalent to $10^y = x$).

Complex Number The sum or difference of a real number and an imaginary number. It is written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit (i.e., $i = \sqrt{-1}$). The a is called the real part, and the bi is called the imaginary part.

Composite Number Any natural number with more than two factors (e.g., 6 is a composite number since it has four factors: 1, 2, 3, and 6). A composite number is not a prime number.

Compound (or Combined) Event An event that is made up of two or more simple events, such as the flipping of two or more coins.

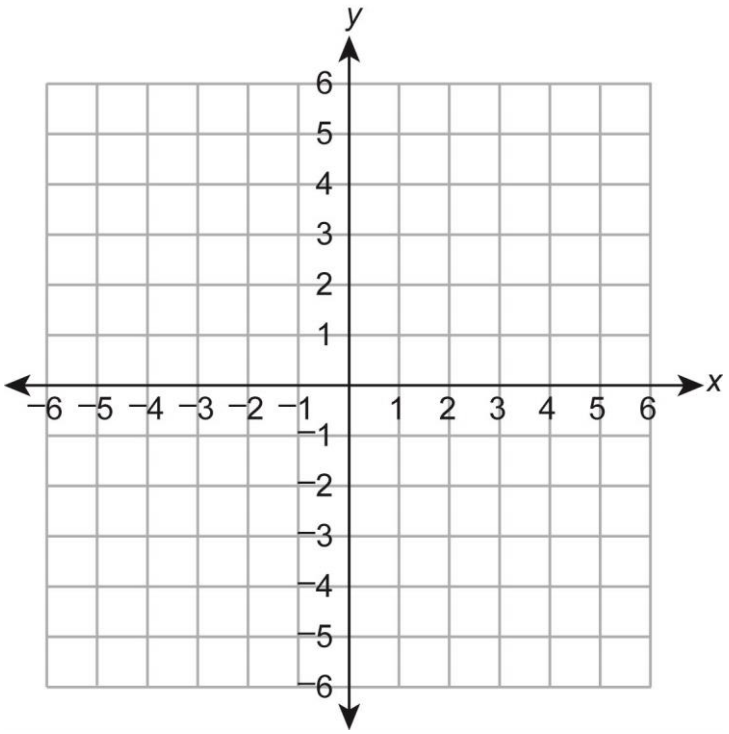
Compound Inequality When two or more inequalities are taken together and written with the inequalities connected by the words *and* or *or* (e.g., $x > 6$ and $x < 12$, which can also be written as $6 < x < 12$).

Constant

A term or expression with no variable in it. It has the same value all the time.

Coordinate Plane

A plane formed by perpendicular number lines. The horizontal number line is the *x*-axis, and the vertical number line is the *y*-axis. The point where the axes meet is called the origin. Example of a coordinate plane:



Cube Root One of three equal factors (roots) of a number or expression; a radical expression with a degree of 3 (e.g., $\sqrt[3]{a}$). The cube root of a number or expression has the same sign as the number or expression under the radical (e.g., $\sqrt[3]{-343x^6} = -(7x^2)$ and $\sqrt[3]{343x^6} = 7x^2$).

Curve of Best Fit (for a Scatter Plot) See line or curve of best fit (for a scatter plot).

Degree (of a Polynomial) The value of the greatest exponent in a polynomial.

Dependent Events Two or more events in which the outcome of one event affects or influences the outcome of the other event(s).

Dependent Variable The output number or variable in a relation or function that depends upon another variable, called the independent variable, or input number (e.g., in the equation $y = 2x + 4$, y is the dependent variable since its value depends on the value of x). It is the variable for which an equation is solved. Its values make up the range of the relation or function.

Domain (of a Relation or Function) The set of all possible values of the independent variable on which a function or relation is allowed to operate. Also, the first numbers in the ordered pairs of a relation; the values of the x-coordinates in (x, y).

Elimination Method See linear combination.

Equation A mathematical statement or sentence that says one mathematical expression or quantity is equal to another (e.g., $x + 5 = y - 7$). An equation will always contain an equal sign (=).

Estimation Strategy An approximation based on a judgment; may include determining approximate values, establishing the reasonableness of answers, assessing the amount of error resulting from estimation, and/or determining if an error is within acceptable limits.

Exponent The power to which a number or expression is raised. When the exponent is a fraction, the number or expression can be rewritten with a radical sign (e.g., $x^{3/4} = \sqrt[4]{x^3}$). See also positive exponent and negative exponent.

Exponential Equation An equation with variables in its exponents (e.g., $4^x = 50$). It can be solved by taking logarithms of both sides.

Exponential Expression An expression in which the variable occurs in the exponent (such as 4^x rather than x^4). Often it occurs when a quantity changes by the same factor for each unit of time (e.g., "doubles every year" or "decreases 2% each month").

Exponential Function (or Model) A function whose general equation is $y = a \cdot b^x$ where a and b are constants.

Exponential Growth/Decay A situation where a quantity increases or decreases exponentially by the same factor over time; it is used for such phenomena as inflation, population growth, radioactivity or depreciation.

Expression	A mathematical phrase that includes operations, numbers, and/or variables (e.g., $2x + 3y$ is an algebraic expression, 13.4 – 4.7 is a numeric expression). An expression does not contain an equal sign (=) or any type of inequality sign.
Factor (noun)	The number or expression that is multiplied by another to get a product (e.g., 6 is a factor of 30, and $6x$ is a factor of $42x^2$).
Factor (verb)	To express or write a number, monomial, or polynomial as a product of two or more factors.
Factor a Monomial	To express a monomial as the product of two or more monomials.
Factor a Polynomial	To express a polynomial as the product of monomials and/or polynomials (e.g., factoring the polynomial $x^2 + x – 12$ results in the product $(x – 3)(x + 4)$).
Frequency	How often something occurs (i.e., the number of times an item, number, or event happens in a set of data).
Function	A relation in which each value of an independent variable is associated with a unique value of a dependent variable (e.g., one element of the domain is paired with one and only one element of the range). It is a mapping which involves either a one-to-one correspondence or a many-to-one correspondence, but not a one-to-many correspondence.

Fundamental Counting Principle

A way to calculate all of the possible combinations of a given number of events. It states that if there are *x* different ways of doing one thing and *y* different ways of doing another thing, then there are *xy* different ways of doing both things. It uses the multiplication rule.

Geometric Sequence

An ordered list of numbers that has the same ratio between consecutive terms (e.g., 1, 7, 49, 343, … is a geometric sequence that has a ratio of 7/1 between consecutive terms; each term after the first term can be found by multiplying the previous term by a constant, in this case the number 7 or 7/1).

Greatest Common Factor (GCF)

The largest factor that two or more numbers or algebraic terms have in common. In some cases the GCF may be 1 or one of the actual numbers (e.g., the GCF of $18x^3$ and $24x^5$ is $6x^3$).

Imaginary Number

The square root of a negative number, or the opposite of the square root of a negative number. It is written in the form bi, where b is a real number and i is the imaginary root (i.e., $i = \sqrt{-1}$ or $i^2 = -1$).

Independent Event(s)

Two or more events in which the outcome of one event does *not* affect the outcome of the other event(s) (e.g., tossing a coin and rolling a number cube are independent events). The probability of two independent events (A and B) occurring is written $P(A \text{ and } B)$ or $P(A \cap B)$ and equals $P(A) \cdot P(B)$ (i.e., the product of the probabilities of the two individual events).

Independent Variable

The input number or variable in a relation or function whose value is subject to choice. It is not dependent upon any other values. It is usually the *x*-value or the *x* in $f(x)$. It is graphed on the *x*-axis. Its values make up the domain of the relation or function.

Inequality A mathematical sentence that contains an inequality symbol (i.e., >, <, ≥, ≤, or ≠). It compares two quantities. The symbol > means greater than, the symbol < means less than, the symbol ≥ means greater than or equal to, the symbol ≤ means less than or equal to, and the symbol ≠ means not equal to.

Integer A natural number, the additive inverse of a natural number, or zero. Any number from the set of numbers represented by {…, $^{-}3$, $^{-}2$, $^{-}1$, 0, 1, 2, 3, …}.

Interquartile Range (of Data) The difference between the first (lower) and third (upper) quartile. It represents the spread of the middle 50% of a set of data.

Inverse (of a Relation) A relation in which the coordinates in each ordered pair are switched from a given relation. The point (*x, y*) becomes (*y, x*), so (3, 8) would become (8, 3).

Irrational Number A real number that cannot be written as a simple fraction (i.e., the ratio of two integers). It is a non-terminating (infinite) and non-repeating decimal. The square root of any prime number is irrational, as are π and e.

Least (or Lowest) Common Multiple (LCM) The smallest number or expression that is a common multiple of two or more numbers or algebraic terms, other than zero.

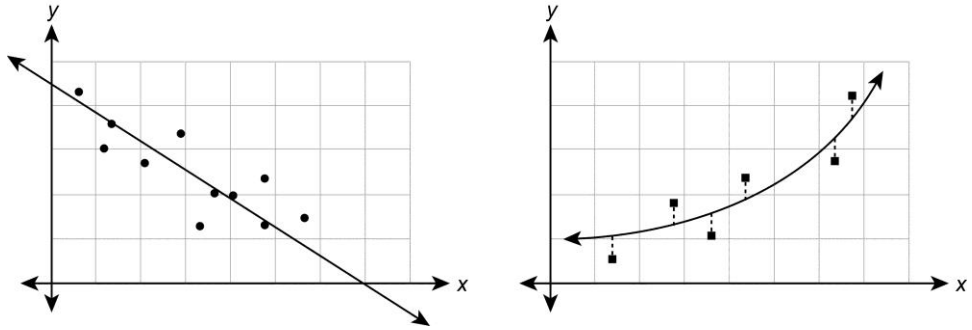
Like Terms Monomials that contain the same variables and corresponding powers and/or roots. Only the coefficients can be different (e.g., $4x^3$ and $12x^3$). Like terms can be added or subtracted.

Line Graph

A graph that uses a line or line segments to connect data points, plotted on a coordinate plane, usually to show trends or changes in data over time. More broadly, a graph to represent the relationship between two continuous variables.

Line or Curve of Best Fit (for a Scatter Plot)

A line or curve drawn on a scatter plot to best estimate the relationship between two sets of data. It describes the trend of the data. Different measures are possible to describe the best fit. The most common is a line or curve that minimizes the sum of the squares of the errors (vertical distances) from the data points to the line. The line of best fit is a subset of the curve of best fit. Examples of a line of best fit and a curve of best fit:



Linear Combination

A method by which a system of linear equations can be solved. It uses addition or subtraction in combination with multiplication or division to eliminate one of the variables in order to solve for the other variable.

Linear Equation

An equation for which the graph is a straight line (i.e., a polynomial equation of the first degree of the form $Ax + By = C$, where A, B, and C are real numbers and where A and B are not both zero; an equation in which the variables are not multiplied by one another or raised to any power other than 1).

Linear Function A function for which the graph is a non-vertical straight line. It is a first degree polynomial of the common form $f(x) = mx + b$, where m and b are constants and x is a real variable. The constant m is called the slope and b is called the y-intercept. It has a constant rate of change.

Linear Inequality The relation of two expressions using the symbols <, >, ≤, ≥, or ≠ and whose boundary is a straight line. The line divides the coordinate plane into two parts. If the inequality is either ≤ or ≥, then the boundary is solid. If the inequality is either < or >, then the boundary is dashed. If the inequality is ≠, then the solution contains everything except for the boundary.

Logarithm The exponent required to produce a given number (e.g., since 2 raised to a power of 5 is 32, the logarithm base 2 of 32 is 5; this is written as $\log_2 32 = 5$). Two frequently used bases are 10 (common logarithm) and e (natural logarithm). When a logarithm is written without a base, it is understood to be base 10.

Logarithmic Equation An equation which contains a logarithm of a variable or number. Sometimes it is solved by rewriting the equation in exponential form and solving for the variable (e.g., $\log_2 32 = 5$ is the same as $2^5 = 32$). It is an inverse function of the exponential function.

Mapping The matching or pairing of one set of numbers to another by use of a rule. A number in the domain is matched or paired with a number in the range (or a relation or function). It may be a one-to-one correspondence, a one-to-many correspondence, or a many-to-one correspondence.

Maximum Value (of a Graph) The value of the dependent variable for the highest point on the graph of a curve.

Mean A measure of central tendency that is calculated by adding all the values of a set of data and dividing that sum by the total number of values. Unlike median, the mean is sensitive to outlier values. It is also called “arithmetic mean” or “average”.

Measure of Central Tendency A measure of location of the middle (center) of a distribution of a set of data (i.e., how data clusters). The three most common measures of central tendency are mean, median, and mode.

Measure of Dispersion A measure of the way in which the distribution of a set of data is spread out. In general the more spread out a distribution is, the larger the measure of dispersion. Range and interquartile range are two measures of dispersion.

Median A measure of central tendency that is the middle value in an ordered set of data or the average of the two middle values when the set has two middle values (occurs when the set of data has an even number of data points). It is the value halfway through the ordered set of data, below and above which there is an equal number of data values. It is generally a good descriptive measure for skewed data or data with outliers.

Minimum Value (of a Graph) The value of the dependent variable for the lowest point on the graph of a curve.

Mode A measure of central tendency that is the value or values that occur(s) most often in a set of data. A set of data can have one mode, more than one mode, or no mode.

Monomial A polynomial with only one term; it contains no addition or subtraction. It can be a number, a variable, or a product of numbers and/or more variables (e.g., $2 \cdot 5$ or x^3y^4 or $\frac{4}{3}\pi r^2$).

Multiplicative Inverse

The reciprocal of a number (i.e., for any non-zero number *a*, the multiplicative inverse is $\frac{1}{a}$; for any rational number $\frac{b}{c}$, where $b \neq 0$ and $c \neq 0$, the multiplicative inverse is $\frac{c}{b}$). Any number and its multiplicative inverse have a product of 1 (e.g., $\frac{1}{4}$ is the multiplicative inverse of 4 since $4 \cdot \frac{1}{4} = 1$; likewise, the multiplicative inverse of $\frac{1}{4}$ is 4 since $\frac{1}{4} \cdot 4 = 1$).

Mutually Exclusive Events

Two events that cannot occur at the same time (i.e., events that have no outcomes in common). If two events A and B are mutually exclusive, then the probability of A or B occurring is the sum of their individual probabilities: $P(A \cup B) = P(A) + P(B)$. Also defined as when the intersection of two sets is empty, written as $A \cap B = \varnothing$.

Natural Logarithm

A logarithm with base e. It is written ln *x*. The natural logarithm is the power of e necessary to equal a given number (i.e., $\ln x = y$ is equivalent to $e^y = x$). The constant e is an irrational number whose value is approximately 2.71828….

Natural Number

A counting number. A number representing a positive, whole amount. Any number from the set of numbers represented by {1, 2, 3, …}. Sometimes, it is referred to as a "positive integer".

Negative Exponent

An exponent that indicates a reciprocal that has to be taken before the exponent can be applied (e.g., $5^{-2} = \frac{1}{5^2}$ or $a^{-x} = \frac{1}{a^x}$). It is used in scientific notation for numbers between $^{-}1$ and 1.

Number Line A graduated straight line that represents the set of all real numbers in order. Typically, it is marked showing integer values.

Odds A comparison, in ratio form (as a fraction or with a colon), of outcomes. "Odds in favor" (or simply "odds") is the ratio of favorable outcomes to unfavorable outcomes (e.g., the odds in favor of picking a red hat when there are 3 red hats and 5 non-red hats is 3:5). "Odds against" is the ratio of unfavorable outcomes to favorable outcomes (e.g., the odds against picking a red hat when there are 3 red hats and 5 non-red hats is 5:3).

Order of Operations Rules describing what order to use in evaluating expressions:
(1) Perform operations in grouping symbols (parentheses and brackets),
(2) Evaluate exponential expressions and radical expressions from left to right,
(3) Multiply or divide from left to right,
(4) Add or subtract from left to right.

Ordered Pair A pair of numbers used to locate a point on a coordinate plane, or the solution of an equation in two variables. The first number tells how far to move horizontally, and the second number tells how far to move vertically; written in the form (*x*-coordinate, *y*-coordinate). Order matters: the point (*x, y*) is **not** the same as (*y, x*).

Origin The point (0, 0) on a coordinate plane. It is the point of intersection for the *x*-axis and the *y*-axis.

Outlier A value that is much greater or much less than the rest of the data. It is different in some way from the general pattern of data. It directly stands out from the rest of the data. Sometimes it is referred to as any data point more than 1.5 interquartile ranges greater than the upper (third) quartile or less than the lower (first) quartile.

Pattern (or Sequence)

A set of numbers arranged in order (or in a sequence). The numbers and their arrangement are determined by a rule, including repetition and growth/decay rules. See arithmetic sequence and geometric sequence.

Perfect Square

A number whose square root is a whole number (e.g., 25 is a perfect square since $\sqrt{25} = 5$). A perfect square can be found by raising a whole number to the second power (e.g., $5^2 = 25$).

Permutation

An ordered arrangement of objects from a given set in which the order of the objects is significant (e.g., two-letter permutations of the three letters X, Y, and Z would be XY, YX, XZ, ZX, YZ, and ZY). A permutation is similar to, but not the same as, a combination.

Point-Slope Form (of a Linear Equation)

An equation of a straight, non-vertical line written in the form $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a given point on the line.

Polynomial

An algebraic expression that is a monomial or the sum or difference of two or more monomials (e.g., $6a$ or $5a^2 + 3a - 13$ where the exponents are natural numbers).

Polynomial Function

A function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where $a_n \neq 0$ and natural number n is the degree of the polynomial.

Positive Exponent

Indicates how many times a base number is multiplied by itself. In the expression x^n, n is the positive exponent, and x is the base number (e.g., $2^3 = 2 \cdot 2 \cdot 2$).

Power

The value of the exponent in a term. The expression a^n is read "*a* to the power of *n*." To raise a number, *a*, to the power of another whole number, *n*, is to multiply *a* by itself *n* times (e.g., the number 4^3 is read "four to the third power" and represents 4 • 4 • 4).

Power of a Power

An expression of the form $(a^m)^n$. It can be found by multiplying the exponents (e.g., $(2^3)^4 = 2^{3•4} = 2^{12} = 4{,}096$).

Powers of Products

An expression of the form $a^m • a^n$. It can be found by adding the exponents when multiplying powers that have the same base (e.g., $2^3 • 2^4 = 2^{3+4} = 2^7 = 128$).

Prime Number

Any natural number with exactly two factors, 1 and itself (e.g., 3 is a prime number since it has only two factors: 1 and 3). [Note: Since 1 has only one factor, itself, it is not a prime number.] A prime number is not a composite number.

Probability

A number from 0 to 1 (or 0% to 100%) that indicates how likely an event is to happen. A very unlikely event has a probability near 0 (or 0%) while a very likely event has a probability near 1 (or 100%). It is written as a ratio (fraction, decimal, or equivalent percent). The number of ways an event could happen (favorable outcomes) is placed over the total number of events (total possible outcomes) that could happen. A probability of 0 means it is impossible, and a probability of 1 means it is certain.

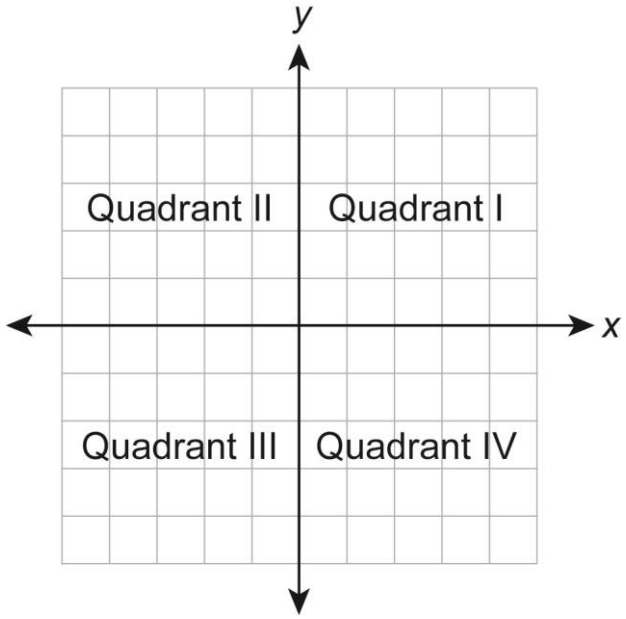
Probability of a Compound (or Combined) Event

There are two types:

1. The union of two events A and B, which is the probability of A *or* B occurring. This is represented as $P(A \cup B) = P(A) + P(B) – P(A) • P(B)$.
2. The intersection of two events A and B, which is the probability of A *and* B occurring. This is represented as $P(A \cap B) = P(A) • P(B)$.

Quadrants

The four regions of a coordinate plane that are separated by the *x*-axis and the *y*-axis, as shown below.



(1) The first quadrant (Quadrant I) contains all the points with positive *x* and positive *y* coordinates (e.g., (3, 4)).
(2) The second quadrant (Quadrant II) contains all the points with negative *x* and positive *y* coordinates (e.g., ($^{-}$3, 4)).
(3) The third quadrant (Quadrant III) contains all the points with negative *x* and negative *y* coordinates (e.g., ($^{-}$3, $^{-}$4)).
(4) The fourth quadrant (Quadrant IV) contains all the points with positive *x* and negative *y* coordinates (e.g., (3, $^{-}$4)).

Quadratic Equation

An equation that can be written in the standard form $ax^2 + bx + c = 0$, where *a*, *b*, and *c* are real numbers and *a* does not equal zero. The highest power of the variable is 2. It has, at most, two solutions. The graph is a parabola.

Quadratic Formula

The solutions or roots of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Quadratic Function

A function that can be expressed in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$ and the highest power of the variable is 2. The graph is a parabola.

Quartile

One of three values that divides a set of data into four equal parts:

1. Median divides a set of data into two equal parts.
2. Lower quartile (25^{th} percentile) is the median of the lower half of the data.
3. Upper quartile (75^{th} percentile) is the median of the upper half of the data.

Radical Expression

An expression containing a radical symbol ($\sqrt[n]{a}$). The expression or number inside the radical (a) is called the radicand, and the number appearing above the radical (n) is the degree. The degree is always a positive integer. When a radical is written without a degree, it is understood to be a degree of 2 and is read as "the square root of a." When the degree is 3, it is read as "the cube root of a." For any other degree, the expression $\sqrt[n]{a}$ is read as "the nth root of a." When the degree is an even number, the radical expression is assumed to be the principal (positive) root (e.g., although $(^{-}7)^2 = 49$, $\sqrt{49} = 7$).

Range (of a Relation or Function)

The set of all possible values for the output (dependent variable) of a function or relation; the set of second numbers in the ordered pairs of a function or relation; the values of the y-coordinates in (x, y).

Range (of Data) In statistics, a measure of dispersion that is the difference between the greatest value (maximum value) and the least value (minimum value) in a set of data.

Rate A ratio that compares two quantities having different units (e.g., $\frac{168\text{ miles}}{3.5\text{ hours}}$ or $\frac{122.5\text{ calories}}{5\text{ cups}}$). When the rate is simplified so that the second (independent) quantity is 1, it is called a unit rate (e.g., 48 miles per hour or 24.5 calories per cup).

Rate (of Change) The amount a quantity changes over time (e.g., 3.2 cm per year). Also the amount a function's output changes (increases or decreases) for each unit of change in the input. See slope.

Rate (of Interest) The percent by which a monetary account accrues interest. It is most common for the rate of interest to be measured on an annual basis (e.g., 4.5% per year), even if the interest is compounded periodically (i.e., more frequently than once per year).

Ratio A comparison of two numbers, quantities or expressions by division. It is often written as a fraction, but not always (e.g., $\frac{2}{3}$, 2**:**3, 2 to 3, 2 ÷ 3 are all the same ratios).

Rational Expression An expression that can be written as a polynomial divided by a polynomial, defined only when the latter is not equal to zero.

Rational Number

Any number that can be written in the form $\frac{a}{b}$ where *a* is any integer and *b* is any integer except zero.

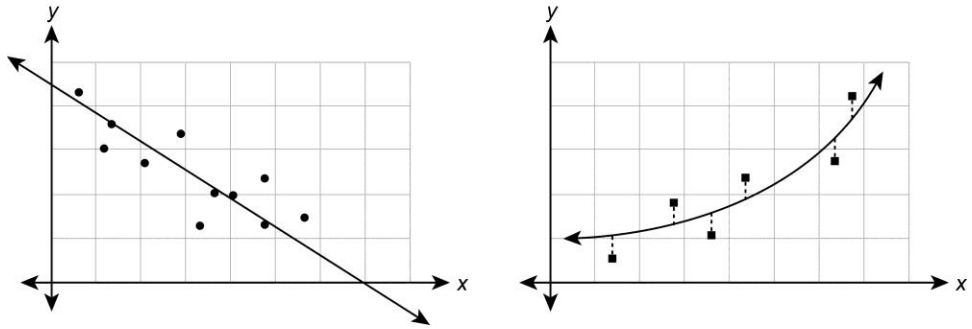
All repeating decimal and terminating decimal numbers are rational numbers.

Real Number

The combined set of rational and irrational numbers. All numbers on the number line. Not an imaginary number.

Regression Curve

The line or curve of best fit that represents the least deviation from the points in a scatter plot of data. Most commonly it is linear and uses a "least squares" method. Examples of regression curves:



Relation

A set of pairs of values (e.g., {(1, 2), (2, 3) (3, 2)}). The first value in each pair is the input (independent value), and the second value in the pair is the output (dependent value). In a relation, neither the input values nor the output values need to be unique.

Repeating Decimal

A decimal with one or more digits that repeats endlessly (e.g., 0.666…, 0.727272…, 0.08333…). To indicate the repetition, a bar may be written above the repeated digits (e.g., 0.666… = $0.\overline{6}$, 0.727272… = $0.\overline{72}$, 0.08333… = $0.08\overline{3}$). A decimal that has either a 0 or a 9 repeating endlessly is equivalent to a terminating decimal (e.g., 0.375000… = 0.375, 0.1999… = 0.2). All repeating decimals are rational numbers.

Rise

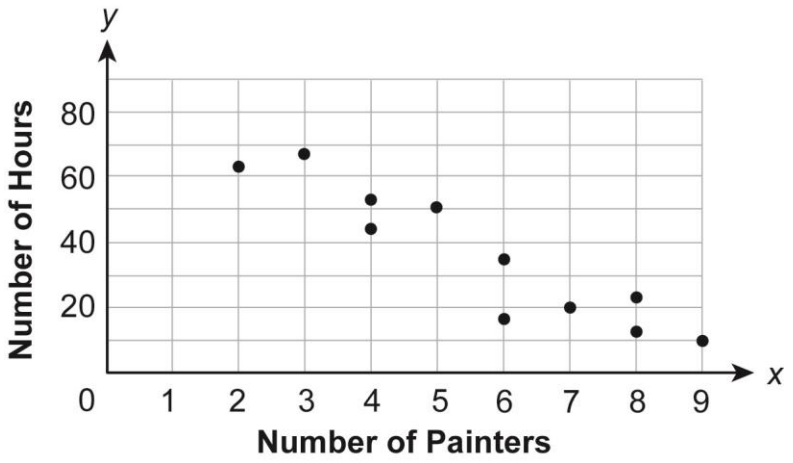
The vertical (up and down) change or difference between any two points on a line on a coordinate plane (i.e., for points (x_1, y_1) and (x_2, y_2), the rise is $y_2 - y_1$). See slope.

Run

The horizontal (left and right) change or difference between any two points on a line on a coordinate plane (i.e., for points (x_1, y_1) and (x_2, y_2), the run is $x_2 - x_1$). See slope.

Scatter Plot

A graph that shows the "general" relationship between two sets of data. For each point that is being plotted there are two separate pieces of data. It shows how one variable is affected by another. Example of a scatter plot:

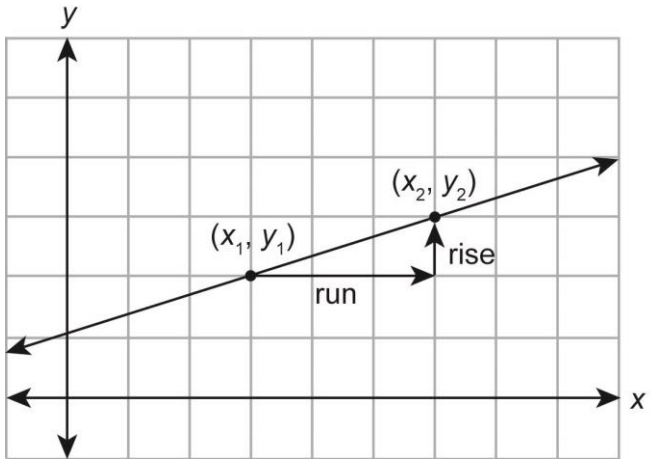


Simple Event When an event consists of a single outcome (e.g., rolling a number cube).

Simplest Form (of an Expression) When all like terms are combined (e.g., $8x + 2(6x – 22)$ becomes $20x – 44$ when in simplest form). The form which no longer contains any like terms, parentheses, or reducible fractions.

Simplify To write an expression in its simplest form (i.e., remove any unnecessary terms, usually by combining several or many terms into fewer terms or by cancelling terms).

Slope (of a Line) A rate of change. The measurement of the steepness, incline, or grade of a line from left to right. It is the ratio of vertical change to horizontal change. More specifically, it is the ratio of the change in the *y*-coordinates (rise) to the corresponding change in the *x*- coordinates (run) when moving from one point to another along a line. It also indicates whether a line is tilted upward (positive slope) or downward (negative slope) and is written as the letter *m* where $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$. Example of slope:



rise = up 1 unit
run = right 3 units

$\text{slope} = \frac{+ 1 \text{ unit}}{+ 3 \text{ units}} = \frac{1}{3}$

Slope-Intercept Form

An equation of a straight, non-vertical line written in the form $y = mx + b$, where m is the slope and b is the y-intercept.

Square Root

One of two equal factors (roots) of a number or expression; a radical expression ($\sqrt{a}$) with an understood degree of 2. The square root of a number or expression is assumed to be the principal (positive) root (e.g., $\sqrt{49x^4} = 7x^2$). The square root of a negative number results in an imaginary number (e.g., $\sqrt{-49} = 7i$).

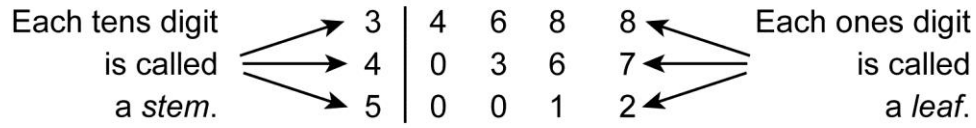
Standard Form (of a Linear Equation)

An equation of a straight line written in the form $Ax + By = C$, where A, B, and C are real numbers and where A and B are not both zero. It includes variables on one side of the equation and a constant on the other side.

Stem-and-Leaf Plot

A visual way to display the shape of a distribution that shows groups of data arranged by place value; a way to show the frequency with which certain classes of data occur. The stem consists of a column of the larger place value(s); these numbers are not repeated. The leaves consist of the smallest place value (usually the ones place) of every piece of data; these numbers are arranged in numerical order in the row of the appropriate stem (e.g., the number 36 would be indicated by a leaf of 6 appearing in the same row as the stem of 3). Example of a stem-and-leaf plot:

Number of Sit-ups



Key
3 | 6 = 36

Substitution The replacement of a term or variable in an expression or equation by another that has the same value in order to simplify or evaluate the expression or equation.

System of Linear Equations A set of two or more linear equations with the same variables. The solution to a system of linear equations may be found by linear combination, substitution, or graphing. A system of two linear equations will either have one solution, infinitely many solutions, or no solutions.

System of Linear Inequalities Two or more linear inequalities with the same variables. Some systems of inequalities may include equations as well as inequalities. The solution region may be closed or bounded because there are lines on all sides, while other solutions may be open or unbounded.

Systems of Equations A set of two or more equations containing a set of common variables.

Term A part of an algebraic expression. Terms are separated by either an addition symbol (+) or a subtraction symbol (–). It can be a number, a variable, or a product of a number and one or more variables (e.g., in the expression $4x^2 + 6y$, $4x^2$ and $6y$ are both terms).

Terminating Decimal A decimal with a finite number of digits. A decimal for which the division operation results in either repeating zeroes or repeating nines (e.g., 0.375000… = 0.375, 0.1999… = 0.2). It is generally written to the last non-zero place value, but can also be written with additional zeroes in smaller place values as needed (e.g., 0.25 can also be written as 0.2500). All terminating decimals are rational numbers.

Trinomial A polynomial with three unlike terms (e.g., $7a + 4b + 9c$). Each term is a monomial, and the monomials are joined by an addition symbol (+) or a subtraction symbol (–). It is considered an algebraic expression.

Unit Rate A rate in which the second (independent) quantity of the ratio is 1 (e.g., 60 words per minute, $4.50 per pound, 21 students per class).

Variable A letter or symbol used to represent any one of a given set of numbers or other objects (e.g., in the equation $y = x + 5$, the y and x are variables). Since it can take on different values, it is the opposite of a constant.

Whole Number A natural number or zero. Any number from the set of numbers represented by {0, 1, 2, 3, …}. Sometimes it is referred to as a "non-negative integer".

***x*-Axis** The horizontal number line on a coordinate plane that intersects with a vertical number line, the *y*-axis; the line whose equation is $y = 0$. The x-axis contains all the points with a zero y-coordinate (e.g., (5, 0)).

***x*-Intercept(s)** The x-coordinate(s) of the point(s) at which the graph of an equation crosses the *x*-axis (i.e., the value(s) of the x-coordinate when $y = 0$). The solution(s) or root(s) of an equation that is set equal to 0.

***y*-Axis** The vertical number line on a coordinate plane that intersects with a horizontal number line, the *x*-axis; the line whose equation is $x = 0$. The y-axis contains all the points with a zero x-coordinate (e.g., (0, 7)).

***y*-Intercept(s)** The y-coordinate(s) of the point(s) at which the graph of an equation crosses the *y*- axis (i.e., the value(s) of the y-coordinate when $x = 0$). For a linear equation in slope-intercept form ($y = mx + b$), it is indicated by b.

Keystone Exams: Algebra II

Assessment Anchors and Eligible Content with Sample Questions and Glossary

April 2014

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