Additional Practice 9-8
Repeated Reasoning

Another Look!

Study each set of problems. Then make a generalization about each set.

Set A

$\frac{1}{4} \div 6 = \frac{1}{24}$ $\frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$

$\frac{1}{3} \div 5 = \frac{1}{15}$ $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

Set B

$6 \div \frac{1}{4} = 24$ $6 \times 4 = 24$

$5 \div \frac{1}{3} = 15$ $5 \times 3 = 15$

> Generalizing can help you find general methods for solving division problems involving unit fractions and whole numbers.

Set A

$\frac{1}{4} \div 6 = \frac{1}{4} \times \frac{1}{6}$
$\frac{1}{3} \div 5 = \frac{1}{3} \times \frac{1}{5}$

Generalization:

Dividing a unit fraction by a whole number other than zero is the same as multiplying the unit fraction by a unit fraction with the whole number as the denominator.
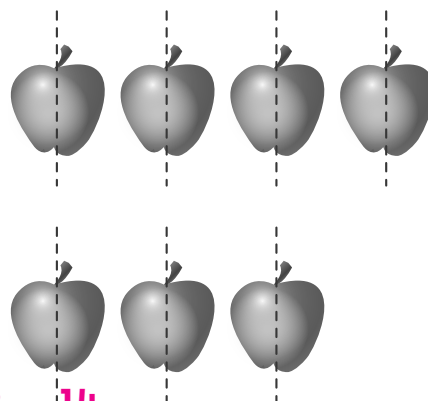
Set B

$6 \div \frac{1}{4} = 6 \times 4$
$5 \div \frac{1}{3} = 5 \times 3$

Generalization:

Dividing a whole number by a unit fraction is the same as multiplying a whole number by the denominator of the unit fraction.

Mrs. Miller brought 7 apples to a picnic. She cut each apple in half. How many pieces did she wind up with?

1. Write and solve a division equation to find the total number of apple pieces. Explain your reasoning.

$7 \div \frac{1}{2} = 14$ pieces; Sample answer: I know that dividing a whole number by a unit fraction is the same as multiplying a whole number by the denominator of the unit fraction, so $7 \div \frac{1}{2} = 7 \times 2 = 14.$

2. Suppose Mrs. Miller decided to cut each apple into fourths rather than into halves. Find how many apple pieces she would have then. Can you repeat the method you used in Exercise 1 to solve this problem? Explain.

28 pieces; Sample answer: Yes, I can write equations like the ones I used in Exercise 1: $7 \div \frac{1}{4} = 7 \times 4 = 28.$

3. When you divide a whole number by a unit fraction, how does the quotient compare to the whole number? Explain.

Sample answer: The quotient is greater than the whole number because to find the quotient, you multiply the whole number by the denominator of the unit fraction, which is greater than 1.
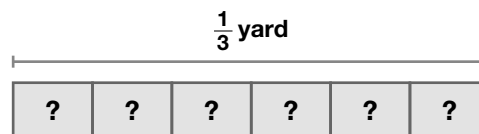
Craft Project

Mariah has the spool of ribbon shown at the right and $\frac{1}{2}$ yard of fabric for a craft project. She wants to cut the ribbon into 6 equal pieces. How long is each piece of ribbon?

$\frac{1}{3}$ yard

?	?	?	?	?	?

4. **Model with Math** Find the length of each ribbon piece in yards. Write a division equation to model the problem.

> $\frac{1}{18}$ yard; $\frac{1}{3} \div 6 = \frac{1}{18}$

5. **Reasoning** Explain how you can use multiplication to check your answer to Exercise 4.

> $\frac{1}{3} \div 6 = \frac{1}{18}$ yard because $\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$.

6. **Be Precise** Find the length of each ribbon piece in inches. Show your work.

> 2 inches; There are 36 inches in 1 yard, so $\frac{1}{18}$ of a yard is $\frac{1}{18} \times 36 = 2$.

7. **Generalize** What generalization can you make that relates the division equation you wrote in Exercise 4 to the multiplication equation you wrote in Exercise 5?

> Sample answer: Dividing a unit fraction by a whole number other than zero is the same as multiplying the unit fraction by a unit fraction with the whole number as the denominator.

Repeated reasoning can help you find shortcuts.

8. **Generalize** Mariah has $\frac{1}{3}$ yard of a gold ribbon. She wants to cut this ribbon into 2 equal pieces. How long is each piece of gold ribbon in yards? Can you repeat the method you used in Exercises 4 and 5 to solve this problem? Explain.

> $\frac{1}{6}$ yard; Sample answer: Yes, I can write division and multiplication equations like the ones I used in Exercises 4 and 5: $\frac{1}{3} \div 2 = \frac{1}{6}$ because $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.