

You must show all of your work to get credit on the test.

✓ If $\sin u = \frac{3}{5}$, $\frac{\pi}{2} < u < \pi$, and $\cos v = \frac{8}{17}$, $\frac{2\pi}{3} < v < 2\pi$, find the exact value of the following.

1. $\sin 2u$
 $2 \sin u \cos u$
 $2 \left(\frac{3}{5} \right) \left(-\frac{4}{5} \right)$
 $= \frac{2}{1} \left(-\frac{12}{25} \right)$
 $= -\frac{24}{25}$

$\frac{3}{5}$ $y=3$
 $r=5$
 $r^2 = x^2 + y^2$
 $25 = x^2 + 9$
 $16 = x^2$
 $x = -4$
 $\cos u = -\frac{4}{5}$

2. $\csc(u-v)$
 $\sin(u-v) = \sin u \cos v - \cos u \sin v$
 $\cos v = \frac{8}{17}$
 $\frac{x}{r} = \frac{8}{17}$
 $x = 8$
 $r = 17$
 $r^2 = x^2 + y^2$
 $(17)^2 = 8^2 + y^2$
 $289 = 64 + y^2$
 $y = 15$
 $\sin v = \frac{15}{17}$
 $\sin(u-v) = \left(\frac{3}{5} \right) \left(\frac{8}{17} \right) - \left(-\frac{4}{5} \right) \left(\frac{15}{17} \right)$
 $= \frac{24}{85} + \frac{60}{85} = \frac{84}{85}$
 $\csc(u-v) = \frac{85}{84}$

3. $\cot 2v$
 $\cot 2v = \frac{1}{\tan 2v} = \frac{1}{\frac{2 \tan v}{1 - \tan^2 v}}$
 $\tan v = \frac{-15}{8}$
 $= \frac{1}{\frac{2 \left(-\frac{15}{8} \right)}{1 - \left(-\frac{15}{8} \right)^2}} = \frac{1}{\frac{-\frac{30}{8}}{\frac{64 - 225}{64}}} = \frac{1}{\frac{-\frac{30}{8}}{-\frac{161}{64}}} = \frac{1}{\frac{30}{161}} = \frac{161}{30}$

Find the exact value of each.

✓ 4. $\sin(75^\circ) = \sin(30^\circ + 45^\circ)$
 $= \sin u \cos v + \cos u \sin v$
 $= \sin 30 \cos 45 + \cos 30 \sin 45$
 $= \left(\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right) + \left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right)$
 $= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$

✓ 5. $\sin(330^\circ) \cos(30^\circ) - \cos(330^\circ) \sin(30^\circ)$
 $= \sin(u-v)$
 $= \sin(330 - 30)$
 $= \sin 300$
 $= -\frac{\sqrt{3}}{2}$

✓ 6. Use power-reducing formulas to write $[\sin^2(x)] \cos^2(x)$ in terms of a first power cosine.

$\frac{1 + \cos 4x}{2}$
 $\left(\frac{1 - \cos 2x}{2} \right) \left(\frac{1 + \cos 2x}{2} \right) = \frac{1 - \cos^2 2x}{4} = \frac{1 - \frac{1 + \cos 4x}{2}}{4} = \frac{\frac{2 - 1 - \cos 4x}{2}}{4} = \frac{1 - \cos 4x}{8}$

✓ 7. Use product-to-sum formulas to write $\cos 2x \sin 4x$ as a sum or difference.

$\cos u \sin v = \frac{1}{2} [\sin(u+v) - \sin(u-v)]$
 $u = 2x$
 $v = 4x$
 $= \frac{1}{2} [\sin(2x+4x) - \sin(2x-4x)]$
 $= \frac{1}{2} [\sin(6x) + \sin(2x)]$
 OR $\frac{\sin(6x) + \sin(2x)}{2}$

✓ 8. Use sum-to-product formulas to find the exact value of $\sin 105^\circ - \sin 15^\circ$

$u = 105$
 $v = 15$
 $\sin u - \sin v = 2 \cos \left(\frac{u+v}{2} \right) \sin \left(\frac{u-v}{2} \right)$
 $= 2 \cos \left(\frac{105+15}{2} \right) \sin \left(\frac{105-15}{2} \right)$
 $= 2 \cos \left(\frac{120}{2} \right) \sin \left(\frac{90}{2} \right)$
 $= 2 \cos(60^\circ) \sin(45^\circ)$
 $= 2 \left(\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right)$
 $= \frac{2}{1} \left(\frac{\sqrt{2}}{4} \right) = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$