Binomial Theorem

Important Concepts:

Factorial

- **Factorial:** the product of an integer and all of the positive integers below it, excluding zero
  - Notation: exclamation point !
  - Ex) $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$
  - Used to compute the number of arrangements possible for a given set of numbers
  - $0! = 1$ (only one way to arrange an empty set)

Important Concepts:

Combinations

- **Combination:** collection of items, in which the order DOES NOT matter

  - Notation: $\binom{n}{r}$ OR ${}_nC_r$ read, "n choose r"

    both equal $\frac{n!}{r!(n-r)!}$ where $n =$ the number of things to choose from

    $r =$ how many we are choosing

  - An example of when the order wouldn't matter …

    You decide to play the lottery and choose a set of numbers. As long as every number is drawn, in any order, you WIN!!!

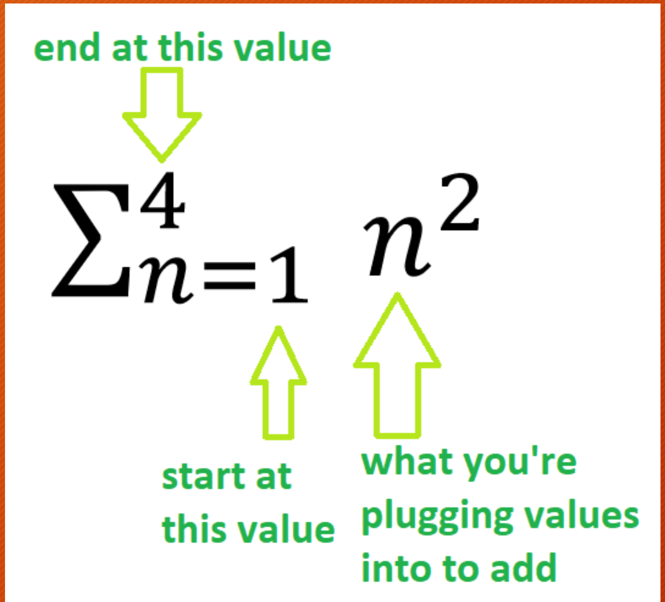
Important Concepts: Combinations (contd.)

- Ex) A group of 5 people are taking a trip. 3 are needed to plan the trip. How many different combinations of 3 people are there?

Important Concepts:

Summation

- **<u>Summation:</u>** the sum of all elements in a sequence
  - Notation: $\sum$
  - Ex) Evaluate $\sum_{n=1}^{4} n^2$



Binomial Expansions

- $(a+b)^0 = 1$
- $(a+b)^1 = a+b$
- $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$
- $(a+b)^3 = (a+b)(a+b)(a+b) = (a+b)(a^2+2ab+b^2) = \ a^3 + 3a^2b + 3ab^2 + b^3$

Exponents on a terms:	3	2	1	0
Exponents on b terms:	0	1	2	3

- $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
- $(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Pattern of exponents: a: 5 → 0 b: 0 → 5

If you know the pattern of the exponents on each variable, the binomial theorem essentially just finds you the coefficients on your terms

If n is the degree of the polynomial, there are n + 1 terms in the expansion

Binomial Theorem

- What if you were asked to simplify $(a+b)^{20}$?
- The **<u>Binomial Theorem</u>** is a quicker way to expand (multiply out) a binomial that has been raised to some power

n = exponent on the binomial

- $$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

k = power of a or b (doesn't matter which, binomial expansions are symmetrical)

This just tells us that we are to add together all of the results we get when plugging numbers in for k

Actual computation we will be preforming

Example a.)

- Expand $(a+b)^5$ using the binomial theorem

Example b.)

- Expand $(2x + 3y)^3$ using the binomial theorem

Example c.)

- Find the 4th term in the expansion $(3x-2)^{10}$

Keep in mind, we want the entire term. This means the coefficient and variables/their exponents!