✓ = Just Like on test

Algebra 2/ Trig IB   Chapter 2 Test Review 2019   Name: Key

Do all work on scrap paper!! There are graphs for #s 13 and 14 on the back you may use.

Complete the square to find the vertex and the axis of symmetry. (state aos) Then, graph the quadratic. Plot at least 5 points including the vertex. You may use your own graph paper. If you scale your graph, make sure to indicate doing so. All pts must fit on graph provided

✓ 1.) $f(x) = 2x^2 + 12x - 4$

$(2x^2 + 12x) - 4$

$2(x^2 + 6x) - 4$

$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$2(x^2 + 6x + 9) - 4 - 18$

$2(x+3)(x+3) - 22$

$2(x+3)^2 - 22$

when we added 9 to the inside, we were really adding a 18 because a 2 is factored out

$\boxed{\text{vertex: } (-3, -22)}$

$\boxed{\text{aos: } x = -3}$

additional points:

x	y
-1	-14
-2	-20

$x = -1$

$f(-1) = 2(-1)^2 + 12(-1) - 4$

$= 2(1) - 12 - 4$

$= 2 - 12 - 4$

$= -10 - 4 = -14$

$x = -2$

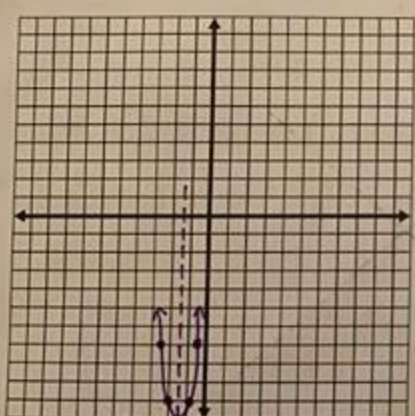
$f(-2) = 2(-2)^2 + 12(-2) - 4$

$= 2(4) - 24 - 4$

$= 8 - 24 - 4$

$= -16 - 4$

$= -20$



scale: by 2's

✓ 2.) $g(x) = x^2 + 4x + 10$

$(x^2 + 4x) + 10$

$\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$(x^2 + 4x + 4) + 10 - 4$

$(x^2 + 4x + 4) + 6$

$(x+2)(x+2) + 6$

$(x+2)^2 + 6$

$\boxed{\text{vertex: } (-2, 6)}$

$\boxed{\text{aos: } x = -2}$

additional points

x	y
0	10
-1	7

$x = 0$

$g(0) = 0^2 + 4(0) + 10$

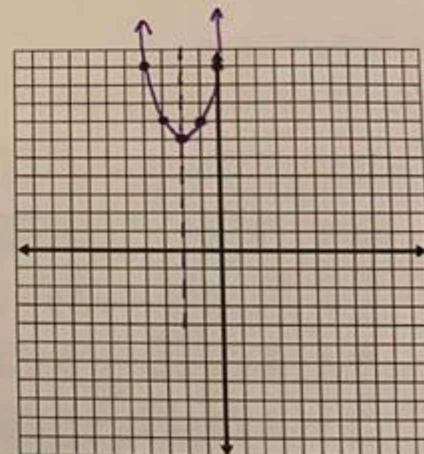
$= 10$

$x = -1$

$g(-1) = (-1)^2 + 4(-1) + 10$

$= 1 - 4 + 10$

$= -3 + 10 = 7$



~~3.)~~ Divide $x^3 - 2x^2 - 9$ by $x - 3$ using synthetic division.

TAKING OFF

$x^3 - 2x^2 + 0x - 9$

3	1	-2	0	-9
	↓	3	3	9
	1	1	3	0

$\boxed{x^2 + x + 3}$

1

✓ 4.) Divide $6x^4 - x^3 - x^2 + 9x - 3$ *by* $x^2 + x - 1$ using polynomial long division.

$$6x^2 - 7x + 12 - \frac{10x+9}{x^2+x-1}$$

$$x^2+x-1 \overline{\big)\, 6x^4 - x^3 - x^2 + 9x - 3}$$

$$-(6x^4 + 6x^3 - 6x^2)$$

$$x^2+x-1 \overline{\big)\, -7x^3 + 5x^2 + 9x - 3}$$

$$-(-7x^3 - 7x^2 + 7x)$$

$$x^2+x-1 \overline{\big)\, 12x^2 + 2x - 3}$$

$$-(12x^2 + 12x - 12)$$

$$-10x + 9$$

Remainder on test (NO place holders)

$$\boxed{6x^2 - 7x + 12 - \frac{10x+9}{x^2+x+1}}$$

Simplify the following.

✓ 5.) $(-2 + \sqrt{-8}) + (5 - \sqrt{-50})$

$$-2 + \sqrt{-8} + 5 - \sqrt{-50}$$

$$3 + \sqrt{4 \cdot -2} - \sqrt{25 \cdot -2}$$

$$3 + \sqrt{4} \cdot \sqrt{-2} - \sqrt{25} \cdot \sqrt{-2}$$

$$3 + 2\sqrt{2 \cdot -1} - 5\sqrt{2 \cdot -1}$$

$$3 + 2\sqrt{2}\sqrt{-1} - 5\sqrt{2}\sqrt{-1}$$

$$3 + 2i\sqrt{2} - 5i\sqrt{2}$$

$$\boxed{3 - 3i\sqrt{2}}$$

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✓ 6.) $(2 + 3i)^2 + (2 - 3i)^2$

$$(2+3i)(2+3i) + (2-3i)(2-3i)$$

$$4 + 6i + 6i + 9i^2 + 4 - 6i - 6i + 9i^2$$

$$8 + 18i^2$$

$$8 + 18(-1)$$

$$8 - 18$$

$$\boxed{-10}$$

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✓ 7.) $-\frac{14}{2i}$

$$-\frac{14}{2i} \cdot \frac{2i}{2i}$$

$$= \frac{-28i}{4i^2} = \frac{-28i}{4(-1)} = \frac{-28i}{-4} = \boxed{7i}$$

✓ 8.) $\frac{6-7i}{1-2i} \cdot \frac{1+2i}{1+2i}$ > SAME TERMS, OPP SIGNS BETWEEN

$$\frac{(6-7i)(1+2i)}{(1-2i)(1+2i)}$$

$$= \frac{6 + 12i - 7i - 14i^2}{1 + 2i - 2i - 4i^2}$$

$$= \frac{6 + 5i - 14(-1)}{1 - 4(-1)} = \frac{6 + 5i + 14}{1+4} = \frac{20 + 5i}{5}$$

$$= \boxed{4 + i}$$

2

polynomial long division or synthetic division to find all of the ***rational*** zeros of the given functions. Then, use them to completely factor the polynomial. You must use synthetic for at least one of these problems. You only need to utilize synthetic division one time in the process. (You may use it more if you wish).

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Include ALL Factors

9.) $t(x) = x^3 - 4x^2 - x + 4$ ← factorable, but you must use synthetic division at least once, so it's easiest to start with it

Possible Rat. zeros: p: 4 → ±1, ±2, ±4; q: 1 → ±1

$\pm\frac{p}{q} = \pm 1, \pm 2, \pm 4$

x = 1 ⟷ factor: (x−1)

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -1 & 4 \\ & \downarrow & 1 & -3 & -4 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

✓

$x^2 - 3x - 4$ → factorable!

$(x-4)(x+1) = 0$

x = 4   x = −1

Rational zeros: $x = \pm 1, 4$

factorization: $(x-4)(x+1)(x-1)$

10.) $p(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

Possible Rational zeros: p: 9 → ±1, ±3, ±9; q: 1 → ±1

$\pm\frac{p}{q} = \pm 1, \pm 3, \pm 9$

x = 1

$$\begin{array}{r|rrrrr} 1 & 1 & 6 & 10 & 6 & 9 \\ & \downarrow & 1 & 7 & 17 & 23 \\ \hline & 1 & 7 & 17 & 23 & 32 \end{array}$$

x = −1

$$\begin{array}{r|rrrrr} -1 & 1 & 6 & 10 & 6 & 9 \\ & \downarrow & -1 & -5 & -5 & -1 \\ \hline & 1 & 5 & 5 & 1 & 8 \end{array}$$

x = 3

$$\begin{array}{r|rrrrr} 3 & 1 & 6 & 10 & 6 & 9 \\ & \downarrow & 3 & 27 & 111 & 351 \\ \hline & 1 & 9 & 37 & 117 & 360 \end{array}$$

x = −3 ⟷ factor: (x+3)

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 10 & 6 & 9 \\ & \downarrow & -3 & -9 & -3 & -9 \\ \hline & 1 & 3 & 1 & 3 & 0 \end{array}$$

✓

$x^3 + 3x^2 + x + 3$ ← factorable!

$(x^3 + 3x^2) + (x+3) = 0$

$x^2(x+3) + (x+3) = 0$

$(x^2+1)(x+3) = 0$

$(x+i)(x-i)(x+3) = 0$

$x = -i$ $x = i$ $x = -3$

Rational zeros: $x = -3$

factorization: $(x+3)(x+3)(x-i)(x+i)$

3

11.) Write $s(x) = x^4 + 6x^2 - 27$ as the product of linear factors, and list **all** of its zeros.

we can factor by grouping!

$x^4 + 6x^2 - 27$

$1 \cdot -27$
$= -27$

use 9 and -3
because $9(-3) = -27$
and
$9 + (-3) = 6 \leftarrow$ coefficient of center term

So, we now have:

$(x^4 - 3x^2) + (9x^2 - 27)$

$x^2(x^2 - 3) + 9(x^2 - 3)$

$(x^2 + 9)(x^2 - 3)$

$\boxed{\text{factorization} \rightarrow (x + 3i)(x - 3i)(x^2 - 3)}$

$\boxed{\text{all zeros: } x = \pm 3i, \pm\sqrt{3}}$

When Factoring
$x^2 + 9 = (x + 3i)(x - 3i)$
OR
$x^2 + 4 = (x + 2i)(x - 2i)$

12.) Write a polynomial function with real coefficients that has the given zeros. 6, $-5 + 2i$

If $-5 + 2i$ is a zero, then $-5 - 2i$ must also be a zero

zeros: $6, -5 + 2i, -5 - 2i$

factors: $(x - 6)(x + 5 - 2i)(x + 5 + 2i)$

$(x - 6)[(x + 5 - 2i)(x + 5 + 2i)]$

$(x - 6)[x^2 + 5x + \cancel{2xi} + 5x + 25 + \cancel{10i} - \cancel{2xi} - \cancel{10i} - 4i^2]$

$(x - 6)(x^2 + 10x + 25 - 4i^2) \rightarrow (x - 6)(x^2 + 10x + 25 - 4(-1))$

$\rightarrow (x - 6)(x^2 + 10x + 25 + 4) \rightarrow (x - 6)(x^2 + 10x + 29) \rightarrow x^3 + 10x^2 + 29x - 6x^2 - 60x - 174$

$\boxed{x^3 + 4x^2 - 31x - 174}$

4

...ch the graph of each rational function. List the x and y intercepts, asymptotes, and test points. These must ...l be included on the graph as well. Finding additional points to plot is optional, however if you choose to find any please list them. (there are graphs on the back of this paper that you can use if needed)

13.) $f(x) = \frac{2x+5}{x-1}$

① y-int:
$x=0$

$y = \frac{2(0)+5}{0-1} = \frac{0+5}{-1} = -5$

$\boxed{\text{y int: } (0,-5)}$

② x-int:
$y=0$

$0 = \frac{2x+5}{x-1}$

$0 = 2x+5$

$-5 = 2x$

$-\frac{5}{2} = x$

$\boxed{\text{xint: } (-2\tfrac{1}{2}, 0)}$

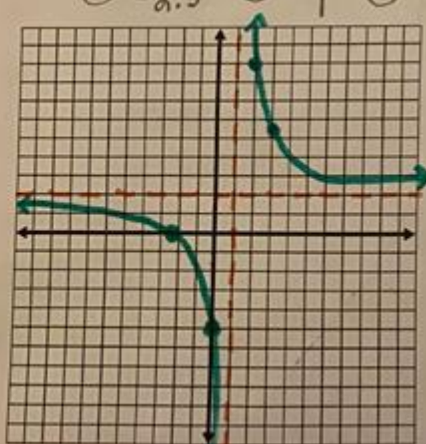
③ vertical asym(s):
$x-1=0 \rightarrow \boxed{x=1}$

④ horizontal asym:
$y = 2/1 \rightarrow \boxed{y=2}$

⑤ Test points:

$+ \quad - \quad +$

$-3 \quad -2.5 \quad 0 \quad 1 \quad 2$



Additional points to guide graphing:

$x=3$

$y = \frac{2(3)+5}{3-1}$

$y = \frac{6+5}{2} = \frac{11}{2} = 5\tfrac{1}{2}$

$(3, 5\tfrac{1}{2})$

we found (2,9) below

So, f(x) is increasing (ABOVE the x axis) on the intervals: $(-\infty,-2.5)$ and $(1,\infty)$
f(x) is decreasing (BELOW the x axis) on the interval $(-2.5,1)$

$x=-3$

$y = \frac{2(-3)+5}{-3-1}$

$y = \frac{-6+5}{-4}$

$y = \frac{-1}{-4} = \frac{1}{4} \leftarrow$ POSITIVE

$x=0$

$y = \frac{2(0)+5}{0-1}$

$y = \frac{0+5}{-1} = \frac{5}{-1} = -5$

NEGATIVE

$x=2$

$y = \frac{2(2)+5}{2-1}$

$y = \frac{4+5}{1} = \frac{9}{1} = 9$

POSITIVE

14.) $k(x) = \frac{x-3}{x^2-3x-10}$

① y-int:
$x=0$

$y = \frac{0-3}{0^2-3(0)-10} = \frac{-3}{-10}$

$\boxed{\text{yint: } (0, 3/10)}$

② x-int:
$y=0$

$0 = \frac{x-3}{x^2-3x-10}$

$0 = x-3 \rightarrow x=3$

$\boxed{\text{X int: } (3,0)}$

③ vertical asym(s):
$x^2-3x-10=0 \rightarrow (x-5)(x+2)=0$

$\boxed{x=5}, \boxed{x=-2}$

④ horizontal asym: $\boxed{y=0}$

⑤ Test points:

$- \quad + \quad - \quad +$

$-3 \quad -2 \quad 0 \quad 3 \quad 4 \quad 5 \quad 6$

So, K(x) is increasing (ABOVE the x axis) on the intervals $(-2,3)$ and $(5,\infty)$ and is decreasing (BELOW the x axis) on the intervals $(-\infty,-2)$ and $(3,5)$

$x=-3$

$y = \frac{-3-3}{(-3)^2-3(-3)-10}$

$y = \frac{-6}{9+9-10} = \frac{-6}{8}$

NEGATIVE

$x=0$

$y = \frac{0-3}{0^2-3(0)-10}$

$y = \frac{-3}{-10} = \frac{3}{10}$

POSITIVE

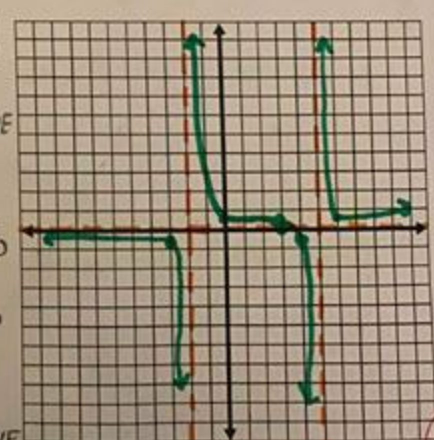
$x=4$

$y = \frac{4-3}{4^2-3(4)-10}$

$y = \frac{1}{16-12-10}$

$y = \frac{1}{-6}$

NEGATIVE



$x=6$

$y = \frac{6-3}{6^2-3(6)-10} = \frac{3}{36-18-10} = \frac{3}{18-10} = \frac{3}{8} \leftarrow$ POSITIVE

5

Solve each nonlinear inequality. Show all work.

15.) $3x^3 - 4x^2 - 12x > -16$

$(3x^3 - 4x^2) - (12x + 16) > 0$

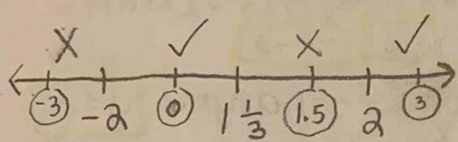
$x^2(3x-4) - 4(3x-4) > 0$

$(x^2-4)(3x-4) > 0$

$(x-2)(x+2)(3x-4) > 0$

$x = 2 \quad x = -2 \quad x = \frac{4}{3} = 1\frac{1}{3}$

critical numbers!



{x = -3}

$3(-3)^3 - 4(-3)^2 - 12(-3) + 16 > 0$

$3(-27) - 36 + 36 + 16 > 0$

$-81 - 36 + 36 + 16 > 0$

$-65 > 0$ false!

{x = 0}

$3(0)^3 - 4(0)^2 - 12(0) + 16 > 0$

$16 > 0$ TRUE!

{x = 3} $3(3)^3 - 4(3)^2 - 12(3) + 16 > 0$

$81 - 36 - 36 + 16 > 0$

$25 > 0$ True!

{x = 1.5}

$3(1.5)^3 - 4(1.5)^2 - 12(1.5) + 16 > 0$

$10.125 - 9 - 18 + 16 > 0$

$-.875 > 0$ false!

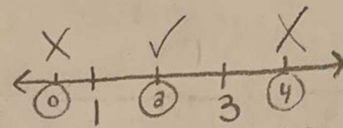
$\boxed{\text{Solutions: } (-2, 1\tfrac{1}{3}) \cup (2, \infty)}$

16.) $\frac{3x-5}{x-3} \leq 1$

$\frac{3x-5}{x-3} - 1 \leq 0 \rightarrow \frac{3x-5}{x-3} - \frac{x-3}{x-3} \leq 0 \rightarrow \frac{3x-5-(x-3)}{x-3} \leq 0$

$\rightarrow \frac{3x-5-x+3}{x-3} \leq 0 \rightarrow \frac{2x-2}{x-3} \leq 0$

Critical numbers: $2x - 2 = 0$, $2x = 2$, $x = 1$ $\quad$ $x - 3 = 0$, $x = 3$



{x = 0}

$\frac{2(0)-2}{0-3} \leq 0$

$\frac{-2}{-3} \leq 0 \rightarrow \frac{2}{3} \leq 0$

FALSE!

{x = 2}

$\frac{2(2)-2}{2-3} \leq 0$

$\frac{4-2}{-1} \leq 0$

$\frac{2}{-1} \leq 0 \rightarrow -2 \leq 0$

TRUE!

{x = 4}

$\frac{2(4)-2}{4-3} \leq 0$

$\frac{8-2}{1} \leq 0$

$6 \leq 0$

FALSE

Now, we must test the critical numbers that are the endpoints of the interval that works! 1 and 3. (They could be included in the interval because the inequality sign is $\leq$)

$x = 1$: $\frac{2(1)-2}{1-3} \leq 0 \rightarrow \frac{0}{-2} \leq 0$, $0 \leq 0$ TRUE!

$x = 3$: $\frac{2(3)-2}{3-3} \leq 0 \quad \frac{4}{0} \leq 0$ FALSE

$\boxed{\text{Solution: } [1, 3)}$

TAKING OFF!

Use Descarte's Rule of Signs to determine the possible number of positive and negative zeros of each function.

17.) $a(x) = 3x^3 + 2x^2 + x + 3$

- positive real roots:

$a(x) = 3x^3 + 2x^2 + x + 3$

No sign changes!

so, $\boxed{\text{No positive real roots}}$

- negative real roots

$a(-x) = 3(-x)^3 + 2(-x)^2 + (-x) + 3$

$= -3x^3 + 2x^2 - x + 3$

6

3 sign changes, so

$\boxed{\text{3 OR 1 negative real root}}$

18.) $j(x) = 2x^4 - 3x + 2$

- positive real roots:

$j(x) = 2x^4 - 3x + 2$

2 sign changes, so

$\boxed{\text{2 or 0 positive real roots}}$

- negative real roots

$j(-x) = 2(-x)^4 - 3(-x) + 2$

$= 2x^4 - 3x + 2$

2 sign changes, so

$\boxed{\text{2 or 0 negative real roots}}$