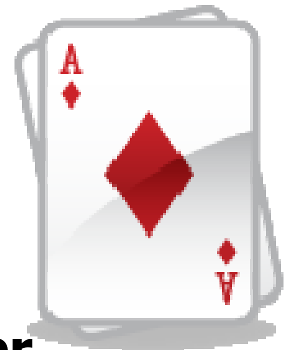
Chapter 15
Random Variables

The Ace of Diamonds



A player pays you $5 and draws a card from the deck. If he draws the ace of diamonds, you pay him $100. For any other ace, you pay $10, and for any other diamond, you pay $5. If he draws anything else, he loses his $5. Who would be willing to play?

What is the expected value? How does the Law of Large Numbers apply? What is the Standard Deviation in this scenario?

Would you play for a top prize of $200?

random variable – its value is based on the outcome of a random event. Denote random variables with capital letters, like X.

discrete random variable – all possible outcomes can be listed

continuous random variable – all possible outcomes cannot be listed

expected value of a discrete random variable – denoted E(X). Multiply each possible value by the probability that it occurs, then find the sum…

$$E(X) = \Sigma(x \bullet P(x))$$

or

the Expected Value is the mean…$E(X) = \mu$

Think of the Expected Value as the Long-Run Average (Long-Run Regularity, Law of Large Numbers, etc.).

be careful…the calculator calls this $\bar{x}$, not μ

<u>standard deviation of a discrete random variable</u> - same as the SD has been earlier in the year, σ

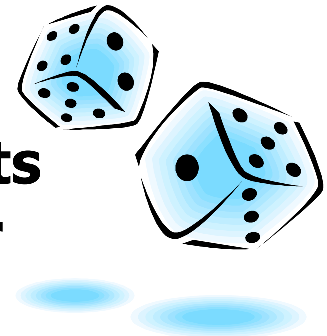
in short:

- **actual sample data have a distribution with a mean $\left(\bar{x}\right)$ and a SD $\left(s\right)$**
- **random variables have a probability model with a mean $\left(\mu\right)$ and a SD $\left(\sigma\right)$**

best news:

let the calculator compute expected values and SDs for you! ☺ (TI Tips)

No Ones, Twos, or Threes



You roll a single die and get no points for rolling a 1, 2, or 3; five points for a 4 or 5; and fifty points for a 6. What are the EV and SD of the game?

Explain, in context, what these values mean.

Now the points awarded are doubled. What are the new mean and SD?

mean ... $E(2X) = 2 \bullet E(X)$

SD ... $SD(2X) = \sqrt{Var(2X)} = \sqrt{2^2 \bullet Var(X)}$

$$= \sqrt{4 \bullet Var(X)} = 2\sqrt{Var(X)}$$

Return to the original point value. Now, you play the game exactly twice. What are your EV and SD?

mean ... $E(X_1 + X_2) = E(X_1) + E(X_2)$

SD ... $SD(X_1 + X_2)$

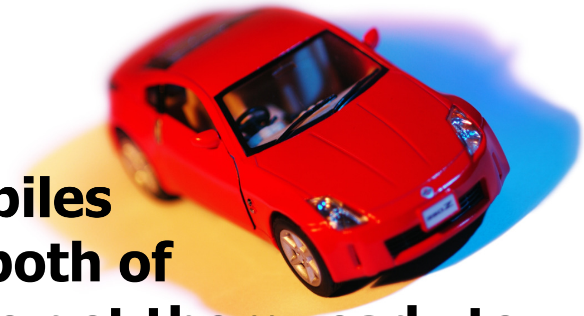
$$= \sqrt{Var(X_1 + X_2)} = \sqrt{Var(X_1) + Var(X_2)}$$

You and a friend both play the game once each. What are the mean and SD of the difference in your winnings?

But why do variances always add…even when you're finding the <u>difference</u> between two random events?

You mix a quart of lemonade. There should be 32 fluid ounces, but your measuring process isn't perfectly accurate, so while 32 ounces is the expected amount, there remains some variability. You pour a 12-ounce glass. Of course, that measurement is not perfect either, so it's actually somewhere around 12, give or take a little. How much is left in the pitcher? Should be around 20 ounces, of course, but given the uncertainty about the initial amount and the variability in how far you filled the glass, you can't say exactly how much is left. Indeed, ***you are less sure*** about the amount left because you removed an unknown amount! Subtracting some lemonade has ***increased*** the variability in the amount remaining! Variances add. **Always.**

Selling a Used Car



A used car dealer runs automobiles through a two-stage process (both of which follow Normal models) to get them ready to sell. The mechanical checkup costs $50 per hour and takes an average of 90 minutes, with a standard deviation of 15 minutes. The appearance prep (wash, vacuum, polish, etc.) costs $6 per hour and takes an average of 60 minutes, with a SD of 5 minutes.

What are the mean and standard deviation of the total time spent preparing a car?

What are the mean and standard deviation of the total expense to prepare a car?

What are the mean and standard deviation of the differences in costs for the two phases of the operation?

What is the probability that it will take longer to do the appearance prep than the mechanical checkup?