

Logarithms and Exponentials

Expanding and Condensing Logarithms

Property	Definition
Product	$\log_b mn = \log_b m + \log_b n$
Quotient	$\log_b \frac{m}{n} = \log_b m - \log_b n$
Power	$\log_b m^p = p \cdot \log_b m$

Ex) Expand the logarithms.

a) $\log_3 9x$

$$\log_3 9 + \log_3 x$$

$$2 + \log_3 x$$

c) $\log_2 8^x$

$$x \log_2 8$$

$$= 3x$$

b) $\log_{20} \frac{y}{400}$

$$\log_{20} y - \log_{20} 400$$

$$\log_{20} y - 2$$

Ex) Condense the logarithms.

a) $\log_3 42 + \log_3 m$

$$\log_3 42m$$

b) $\log_7 x - \log_7 y$

$$\log_7 \frac{x}{y}$$

c) $7 \log_p n$

$$\log_p n^7$$

Evaluating logarithms at given values

Ex) Expand the logarithms and then evaluate them at the given values.

✓ a) $\log_a xy$ if $\log_a x = 5$ $\log_a y = 7$

$$\log_a x + \log_a y$$

$$5 + 7 = 12$$

✓ b) $\log_b \frac{m^2}{n^3}$ if $\log_b m = 4$ $\log_b n = 3$

$$\log_b m^2 - \log_b n^3$$

$$2 \log_b m - 3 \log_b n$$

$$2(4) - 3(3) = 8 - 9 = -1$$

Solving Logarithmic Equations

1. Isolate the logarithm part of the equation (this may involve condensing using properties of logs if there is more than one log term)
2. Rewrite the log using the property $\log_a x \rightarrow a^{\log_a x}$ to get the variable out of the log
3. Solve the remaining equation for the variable

Ex) Solve $6 + \log_5(x+1) = 8$

$$\log_5(x+1) = 2$$

$$5^{\log_5(x+1)} = 5^2$$

$$x+1 = 25$$

$$x = 24$$

✓ Ex) Solve $\log_3(x+2) - \log_3 \frac{1}{3} = 4$

$$\log_3 \frac{1}{3} = -1, \text{ so}$$

$$\log_3(x+2) - (-1) = 4$$

$$\log_3(x+2) + 1 = 4$$

$$\log_3(x+2) = 3$$

$$3^{\log_3(x+2)} = 3^3$$

$$x+2 = 27$$

$$x = 25$$