**AP Stats**
**Chap 17**

Name _____ Pd _____

# Randomness Is Streakier Than We Think


How can we recognize a streak of unusually successful coin tosses?

People have many misconceptions about Bernoulli trials. Often people do not realize how long the longest streak of successes (or failures) in a sequence of Bernoulli trials typically is – usually longer than we think. Consequently, many people feel the need to explain the streaks by abandoning the notion that they are actually observing independent trials. They speak of lottery numbers, gambling tables, and sports players as being "hot" or "cold." This activity will attempt to demonstrate (and debunk) the common misconception known as **The Law of Averages**, as well as attempt to determine the expected value of the longest length of run of successive heads in a sequence of 200 coin flips.

**1.** Of the following two sequences, one is the result from actually tossing a fair coin. The other was made up by a person. Which one do you think is from the coin?

THTTTTHHTHTHTTTTTHHTHTHHHHHHHHHHTHTHTHHHTHHHTTHHTHT
HHHHHTHHTHHTTHTHTHTTTTTTHHTHTHTHTTTHTTTTHHTTTTHTTTT
HTTTTHHHTHTHHTTTHTTHTTHTTHHTHHHTHHHHHTHHHTTHHHTHHT
TTTTTTHTTTHTTTHTTHTTTHTTHHHHTHHHTTTHTTTTTTHHTHTTTHHT

THTHTTTHTTTTHHTHTTTHTTHHHTHHTHTHTHTTTTHHTTTHHTTHHHT
HHHTTHHHTTTHHHTHHHHTTTHTHTHHHHTHTTHHHTHHTHTTTHHTH
HHTHHHHTTHTHHTHHHTTTHTHHHTHHTTTHHHTTTTHHHTHTHHHHT
HTTHHTTTTHTHTHTTHTHHTTHTTTHTTTTHHHHTHTHHHTTHHTHHTH

Why?

**2.** Flip the penny 200 times, keeping record of the results.

| **Flip #** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Result** | | | | | | | | | | | | | | | | | | | | |
| **Flip #** | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| **Result** | | | | | | | | | | | | | | | | | | | | |
| **Flip #** | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| **Result** | | | | | | | | | | | | | | | | | | | | |
| **Flip #** | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| **Result** | | | | | | | | | | | | | | | | | | | | |
| **Flip #** | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| **Result** | | | | | | | | | | | | | | | | | | | | |

| Flip # | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Result | | | | | | | | | | | | | | | | | | | | |
| Flip # | 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 | 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 |
| Result | | | | | | | | | | | | | | | | | | | | |
| Flip # | 141 | 142 | 143 | 144 | 145 | 146 | 147 | 148 | 149 | 150 | 151 | 152 | 153 | 154 | 155 | 156 | 157 | 158 | 159 | 160 |
| Result | | | | | | | | | | | | | | | | | | | | |
| Flip # | 161 | 162 | 163 | 164 | 165 | 166 | 167 | 168 | 169 | 170 | 171 | 172 | 173 | 174 | 175 | 176 | 177 | 178 | 179 | 180 |
| Result | | | | | | | | | | | | | | | | | | | | |
| Flip # | 181 | 182 | 183 | 184 | 185 | 186 | 187 | 188 | 189 | 190 | 191 | 192 | 193 | 194 | 195 | 196 | 197 | 198 | 199 | 200 |
| Result | | | | | | | | | | | | | | | | | | | | |

**3.** Complete the frequency table for the lengths of the "runs" of heads…meaning, one-in-a-row, two-in-a-row, three-in-a-row, etc.

| Length of Run | Frequency |
|---|---|
| 1-in-a-row | |
| 2-in-a-row | |
| 3-in-a-row | |
| 4-in-a-row | |
| 5-in-a-row | |
| 6-in-a-row | |
| 7-in-a-row | |
| 8-in-a-row | |
| 9-in-a-row | |
| 10-in-a-row | |

**4.** Look at a histogram of the data.

- Open the Excel document "Randomness Is Streakier Than We Think."
- **IMMEDIATELY** save it to your H drive using the same file name.
- Close the original file.
- Open the file that you just copied into your H drive.
- Under the "Frequency" column, replace the zeros with your group's numbers. If you had a length longer than 10, ask for assistance! ☺
- As you enter the frequencies, you should see that the **Relative Frequencies** are being calculated and the histogram is being constructed.

**5.** On a sheet of loose leaf – in pencil – answer the following questions.

- Describe the shape of the histogram. How close to a Normal model is it?
- Use your calculator to find the expected value and standard deviation of this data.
- Do these two values **support** your answer to Question 1 above, or would you like to change your mind? Completely explain your position using statistical evidence.
- What is your longest run of heads? Be prepared to share this number with the class.

# Bernoulli Trials

**The longest run of a repeated outcome (success or failure) for *n* Bernoulli trials with probability of success *p* and probability of failure *q* is approximately**

$$\log_{\frac{1}{p}}(nq)$$