7-2 Study Guide and Intervention

Division Properties of Exponents

Divide Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a , $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b , $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 Simplify $\frac{a^4b^7}{ab^2}$. Assume

that no denominator equals zero.

$$\dfrac{a^4b^7}{ab^2}=\dfrac{\left(a^4\right)\!\left(b^7\right)}{\left(b^2\right)}$$
 Group powers with the same base.
$$=(a^{4-1})(b^{7-2})$$
 Quotient of Powers
$$=a^3b^5$$
 Simplify.

The quotient is a^3b^5 .

Example 2 Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume

that no denominator equals zero.

$$\begin{split} \left(\frac{2a^3b^5}{3b^2}\right)^3 &= \frac{(2a^3b^5)^3}{(3b^2)^3} & \text{Power of a Quotient} \\ &= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3} & \text{Power of a Product} \\ &= \frac{8a^9b^{15}}{27b^6} & \text{Power of a Power} \\ &= \frac{8a^9b^9}{27} & \text{Quotient of Powers} \end{split}$$
 The quotient is $\frac{8a^9b^9}{27}$.

Simplify each expression. Assume that no denominator equals zero.

1.
$$\frac{5^5}{5^2}$$

2.
$$\frac{m^6}{m^4}$$

3.
$$\frac{p^5n^4}{p^2n}$$

4.
$$\frac{a^2}{a}$$

5.
$$\frac{x^5y^3}{x^5y^2}$$

6.
$$\frac{-2y^7}{14y^5}$$

7.
$$\frac{xy^6}{y^4x}$$

8.
$$\left(\frac{2a^2b}{a}\right)^3$$

9.
$$\left(\frac{4p^4r^4}{3p^2r^2}\right)^3$$

10.
$$\left(\frac{2r^5w^3}{r^4w^3}\right)^4$$

11.
$$\left(\frac{3r^6n^3}{2r^5n}\right)^4$$

12.
$$\frac{r^7n^7t^2}{n^3r^3t^2}$$

Lesson 7-2

Study Guide and Intervention (continued) 7-2

Division Properties of Exponents

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^{\circ} = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents.

Zero Exponent	For any nonzero number a , $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n , $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that no denominator equals zero.

$$\begin{array}{ll} \frac{4a^{-3}b^6}{16a^2b^6c^{-5}} &= \left(\frac{4}{16}\right)\!\!\left(\frac{a^{-3}}{a^2}\right)\!\!\left(\frac{b^6}{b^6}\!\!\right)\!\!\left(\frac{1}{c^{-5}}\right) & \text{Group powers with the same base.} \\ &= \frac{1}{4}\left(a^{-3-2}\right)\!\!\left(b^{6-6}\right)\!\!\left(c^5\right) & \text{Quotient of Powers and Negative Exponent Properties} \\ &= \frac{1}{4}\,a^{-5}b^0c^5 & \text{Simplify.} \\ &= \frac{1}{4}\!\left(\frac{1}{a^5}\!\!\right)\!\!\left(1\right)\!c^5 & \text{Negative Exponent and Zero Exponent Properties} \\ &= \frac{c^5}{4a^5} & \text{Simplify.} \end{array}$$

The solution is $\frac{c^5}{4a^5}$.

Exercises

Simplify each expression. Assume that no denominator equals zero.

1.
$$\frac{2^2}{2^{-3}}$$

2.
$$\frac{m}{m^{-4}}$$

3.
$$\frac{p^{-8}}{p^3}$$

4.
$$\frac{b^{-4}}{b^{-5}}$$

$$5. \frac{\left(-x^{-1}y\right)^0}{4w^{-1}y^2}$$

6.
$$\frac{(a^2b^3)^2}{(ab)^{-2}}$$

7.
$$\frac{x^4y^0}{x^{-2}}$$

8.
$$\frac{(6a^{-1}b)^2}{(b^2)^4}$$

9.
$$\frac{(3rt)^2u^{-4}}{r^{-1}t^2u^7}$$

$$10. \ \frac{m^{-3}t^{-5}}{(m^2t^3)^{-1}}$$

11.
$$\left(\frac{4m^2n^2}{8m^{-1}\ell}\right)^0$$

12.
$$\frac{(-2mn^2)^{-3}}{4m^{-6}n^4}$$