1.7 Transformations of Functions

There are 2 categories of graph transformations: **rigid:**

and **non-rigid**:

Rigid transformation: Vertical and Horizontal Shifts

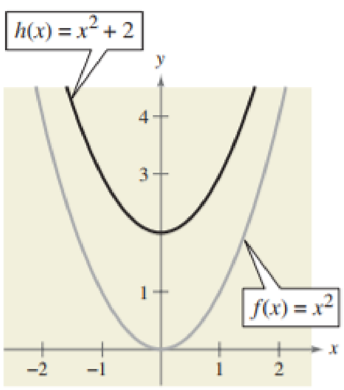
Many functions have graphs that are a simple shift of their parent function (those covered in 1.6).

Vertical and Horizontal Shifts

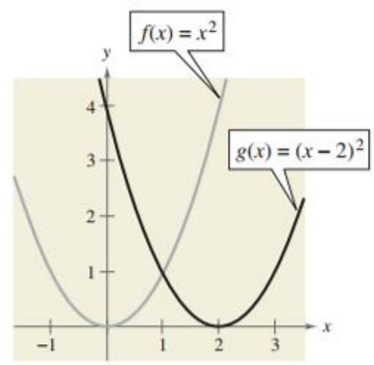
Let c be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.

1. Vertical shift c units *upward:* $\quad h(x) = f(x) + c$
2. Vertical shift c units *downward:* $\quad h(x) = f(x) - c$
3. Horizontal shift c units to the *right:* $\quad h(x) = f(x - c)$
4. Horizontal shift c units to the *left:* $\quad h(x) = f(x + c)$

Ex 1) To obtain the graph of $h(x) = x^2 + 2$ would shift the graph of $f(x) = x^2$ *upward* 2 units



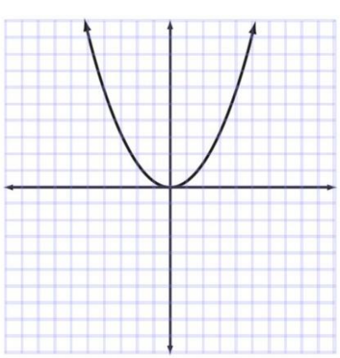
Ex 2) To obtain the graph of $g(x) = (x - 2)^2$ we shift the graph of $f(x) = x^2$ *to the right* 2 units

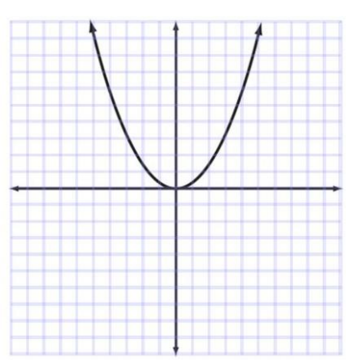


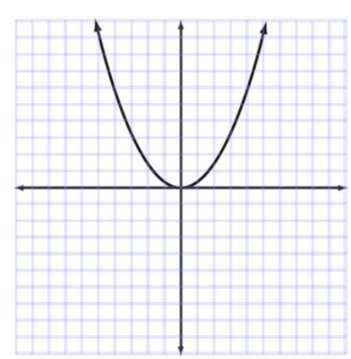
Some graphs can be obtained from combinations of vertical and horizontal shifts. *When graphing utilizing these shifts, it does not matter if you shift horizontally or vertically first. (this is true if the **only** transformations are horizontal and/or vertical)*

Examples: Describe how the graph of each function is shifted compared to the graph of its parent function. (Roughly) sketch the function on the same graph of its parent function.

a) $f(x) =$ b.) $g(x) =$ c.) $h(x) =$







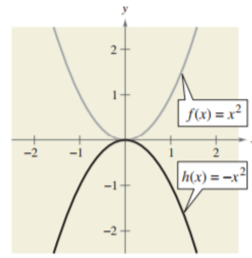
Rigid transformation: Reflections

A 2nd common type of transformation is a reflection. For example, if you consider the x-axis to be a mirror, the graph of $h(x) = -x^2$ is the mirror image, or reflection, of the graph of $(x) = x^2$, as shown below to the right.
Reflections must be done before horizontal and/or vertical shifts when graphing using transformations.

Reflections in the Coordinate Axes

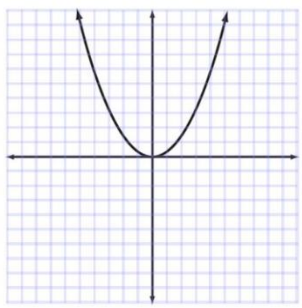
Reflections in the coordinate axes of the graph of $y = f(x)$ are represented as follows.

1. Reflection in the x-axis: $h(x) = -f(x)$
2. Reflection in the y-axis: $h(x) = f(-x)$

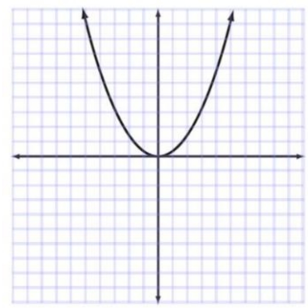


Examples: Describe how the graph of each function is shifted compared to the graph of its parent function. (Roughly) sketch the function on the same graph of its parent function.

d.) $k(x) =$

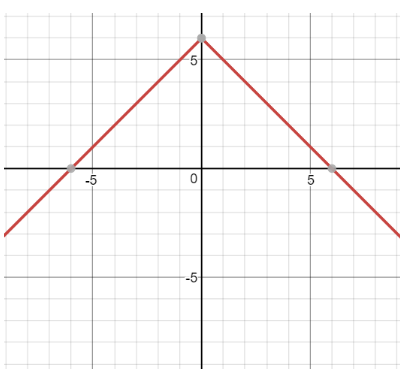


e.) $m(x) =$

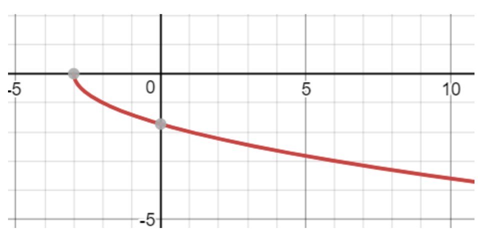


Examples: Compare the graphs of f, g and h with the graph of their parent functions below each. Write each function & state domain and range.

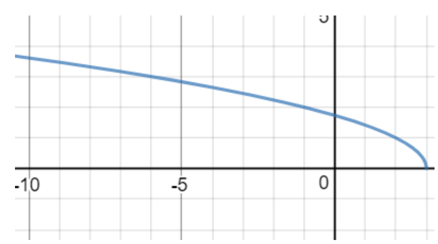
f.)



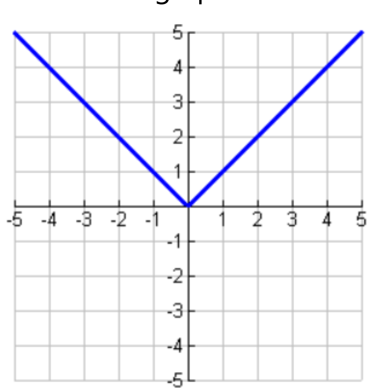
g.)



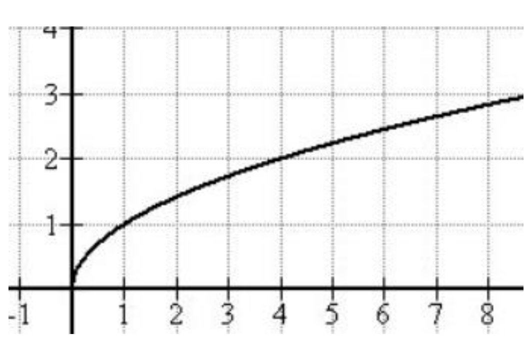
h.)



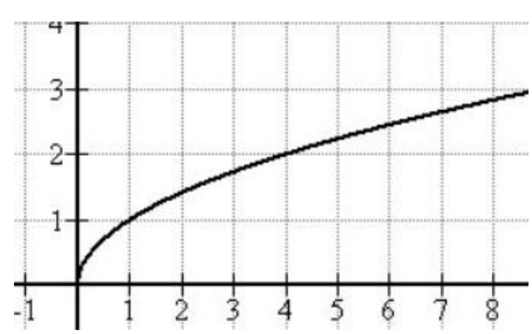
Parent graph:



Parent graph:

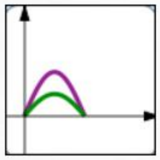
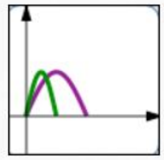
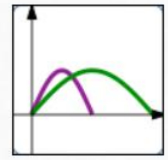


Parent graph:



Non-Rigid transformation: stretches and shrinks

• Vertical stretch	$cf(x),\ c > 1$	Ex) $f(x) = 2x^2$
• Vertical shrink	$cf(x),\ 0 < c < 1$	Ex) $g(x) = \frac{1}{3}\lvert x\rvert$
• Horizontal shrink	$f(cx),\ c > 1$	Ex) $k(x) = (6x)^3$
• Horizontal stretch	$f(cx),\ 0 < c < 1$	Ex) $m(x) = \left(\frac{1}{2}x\right)^3$
Where c is a constant		

			
$y = f(x)$ $y = 2f(x)$ vertical stretch; y-values are doubled; points get farther away from x-axis	$y = f(x)$ $y = \frac{f(x)}{2}$ vertical shrink; y-values are halved; points get closer to x-axis	$y = f(x)$ $y = f(2x)$ horizontal shrink; x-values are halved; points get closer to y-axis	$y = f(x)$ $y = f(\frac{x}{2})$ horizontal stretch; x-values are doubled; points get farther away from y-axis

Here is the thought process you should use when asked about (for example) the graph of $y = 3f(x)$

1. Consider the original equation/graph of the parent function
2. Consider the new equation, $y = 3f(x)$
3. Interpretation of new equation:

$$\overbrace{y}^{\text{the new y-values}} \quad \overbrace{=}^{\text{are}} \quad \overbrace{3}^{\text{three times}} \quad \overbrace{f(x)}^{\text{the previous y-values}}$$

**** Remember, unless you are asked to graph a function in a specific way, which you rarely will be unless its linear, the best bet is to create a chart of x and y values, and find points to plot by choosing x values to plug in and solve to get y values. It is even easier to graph functions if you know the graphs of their parent functions, as well as the transformations. ******

Order in Which to Preform Transformations of Graphs

If you are graphing a function based upon translations of its parent function, here is the order in which to perform the transformations (if there are many):

Steps for a Sequence of Transformations

Apply the following steps when graphing by hand a function containing more than one transformation. Apply the transformations in this order:

1. Start with parentheses (look for possible horizontal shift)
(This could be a vertical shift if the power of x is not 1.)

2. Deal with multiplication (stretch or compression)

3. Deal with negation (reflection)

4. Deal with addition/subtraction (vertical shift)

These steps are basically just following *order of operations*.
(parenthesis, mult/divide, negation, add/subtract)

As stated earlier, if the only transformations are horizontal and vertical, it does not matter which order you preform each. If there are horizontal/vertical shifts and reflections, reflect first. If the combination differs from these, refer to the chart above.