Absolute Value

Solving equations with absolute value

1. Isolate the absolute value part of the equation
2. Set the quantity inside the absolute value equal to $+$ and – the quantity on the other side of the equals sign
3. Solve each for the variable
4. Check your answers by plugging them back into the equation
5. Plug your answers back into the original equation to check that they are solutions that work.

Ex) Solve $|2x - 1| = 5$

Ex) Solve $4 + 2|3 + 3x| = 28$

Finding x and y intercepts of equations with absolute value

- y intercept → set x equal to 0, solve for y. $(0, y)$
- x intercept → set y equal to 0, solve for x using rules above. $(x, 0)$

Ex) Find the x and y intercepts of a) $y = |2x + 5| - 3$ b) $y = -|x + 10|$

Radian and Degree Conversions

Converting from degrees to radians: multiply the degree measure by $\frac{\pi}{180}$

Ex) Convert 60° to radians

Converting from radians to degrees: multiply the radian measure by $\frac{180}{\pi}$

Ex) Convert $\frac{3\pi}{4}$ to degrees

Logarithms and Exponentials

Expanding and Condensing Logarithms

Property	Definition
Product	$\log_b mn = \log_b m + \log_b n$
Quotient	$\log_b \frac{m}{n} = \log_b m - \log_b n$
Power	$\log_b m^p = p \cdot \log_b m$

Ex) Expand the logarithms.

a) $\log_3 9x$

b) $\log_{20} \frac{y}{400}$

c) $\log_2 8^x$

Ex) Condense the logarithms.

a) $\log_3 42 + \log_3 m$

b) $\log_7 x - \log_7 y$

c) $7 \log_p n$

Evaluating logarithms at given values

Ex) Expand the logarithms and then evaluate them at the given values.

a) $\log_a xy \quad if \quad \log_a x = 5 \quad \log_a y = 7$

b) $\log_b \frac{m^2}{n^3} \quad if \quad \log_b m = 4 \quad \log_b n = 3$

Solving Logarithmic Equations

1. Isolate the logarithm part of the equation (this may involve condensing using properties of logs if there is more than one log term
2. Rewrite the log using the property $\log_a x \;\rightarrow\; a^{\log_a x}$ to get the variable out of the log
3. Solve the remaining equation for the variable

Ex) Solve $6 + \log_5(x + 1) = 8$

Ex) Solve $\log_3(x + 2) - \log_3 \frac{1}{3} = 4$

Solving Exponential Equations

1. Isolate the exponential part of the equation (the part that has a variable in the exponent)
2. Rewrite the equation by "taking the ln of both sides" using the property $a^x \rightarrow \ln(a^x) \rightarrow x \ln a$ to get the variable out of the exponent
3. Solve the remaining equation for the variable, using your calculator to evaluate any ln terms

Ex) Solve $3^{x+2} = 18$

Ex) $5^{2x} = \frac{1}{5^x}$

Ex) $1 + 6^{2x-1} = 10$

Solving Other Kinds of Equations

Polynomials

- Quadratics $(ax^2 + bx + c) \rightarrow$ factor or use the quadratic formula
- Polynomials with no powers larger than 1 $\rightarrow$ get x by itself on one side!

Solve the following for x.

a) $x^2 - 2x - 3 = 0$

b) $x^2 + x - 4 = 0$

c) $-5(1 + 3x) = -24 + 4x$

d) $3x^2 + 6x + 6 = 0$

Equations Involving Square Roots

** Make sure to plug your answers back into the original equation to make sure it is an actual solution!!!**

Ex) Solve $x - 3 = \sqrt{x + 3}$

Ex) Solve $\sqrt{2x + 3} = x + 2$

Ex) Solve $x + 6 = \sqrt{7x + 86}$

Rational Equations (equations with fractions)

If/once you have ONE fraction on each side, you can cross multiply to solve for the variable.

** Make sure to plug your answers back into the original equation to make sure it is an actual solution!!!**

Ex) Solve $\frac{x-2}{2x} = \frac{1}{x}$

Ex) Solve $\frac{1}{x} = \frac{x-2}{x^2-5x} + \frac{1}{x-5}$

Ex) $\frac{4}{x^2+4x} + \frac{1}{x} = \frac{1}{x^2+4x}$

Zeros of Polynomials

Writing a Function Given the Zeros

Ex) Write a function with the given the zeros. -1, 1, 6

If a polynomial has zeros, then you can write:

- $x = -1$ $(x+1)$
- $x = 1$ $(x-1)$
- $x = 6$ $(x-6)$

So, we have $(x+1)(x-1)(x-6) = x^3 - 6x^2 - x + 6$

Ex) Write a function with the given the zeros.

a) $5,\ -4,\ 1$

b) $2, -2, 3$

Finding/Writing the Equation of a Line

Given a Point the Line Passes Through and a Line it is Perpendicular to

1. Find the slope of the given line, and use this to get the slope of the new line – it is the opposite reciprocal
2. Use this new slope and the given point to plug into the equation $y - y_1 = m(x - x_1)$ OR $y = mx + b$ and solve this for y to get it into the form $y = mx + b$

Ex) Find the equation of the line that passes through the point (7,2) and is perpendicular to $3x - 2y = 6$

Ex) Find the equation of the line that passes through the point (-2, 6) and is perpendicular to $3x + 6y = 12$

Given a Point the Line Passes Through and a Line it is Parallel to

1. Find the slope of the given line, and use this to get the slope of the new line – they have the same slope
2. Use this new slope and the given point to plug into the equation $y - y_1 = m(x - x_1)$ OR $y = mx + b$ and solve this for y to get it into the form $y = mx + b$

Ex) Find the equation of the line that passes through the point $(1, 7)$ and is parallel to $y - 3x = 5$

Ex) Find the equation of the line that passes through the point $(5, 9)$ and is parallel to $y - 4x = 3$

Operations on Functions

$(f + g)(x) = f(x) + g(x)$ $(f - g)(x) = f(x) - g(x)$ $(fg)(x) = f(x) \bullet g(x)$

$(f \circ g)(x) = f\big(g(x)\big)$ $(g \circ f)(x) = g(f(x)$ $(f \circ f^{-1})(x) = f(f^{-1}(x))$

$f^{-1}(x)$ [the inverse of $f(x)$] is obtained by:

1. Replace $f(x)$ with y
2. Switch x and y
3. Solve the equation for x
4. Replace y with f^{-1}

Ex) Given $f(x) = 3x + 2$ and $g(x) = 4 - 5x$ find the following.

a) $(f + g)(x)$ b) $(f - g)(x)$ c) $(fg)(x)$

d) $(f \circ g)(x)$

e) $(g \circ f)(x)$

f) $f^{-1}(x)$

g) $(f \circ f^{-1})(x)$

Finding Zeros of a Polynomial

Rational Zeros Test to Find the Possible Zeros

Possible rational zeros: $\frac{p}{q}$ where $p =$ factors of the constant term and $q =$ factors of the leading coefficient

Testing the Possible Zeros

It is easiest to first plug in the easy numbers, like 1 and -1, into the function to see if the answer is zero, meaning it is in turn a zero of the polynomial. Once you have found a rational zero, you can use synthetic division to break the polynomial down and find the remaining zeros.

Once you have broken the polynomial down into a 4 term polynomial, you can factor by grouping to continue to solve.

Ex) Find the zeros of $x^4 - x^3 + x^2 - 3x - 6$

Ex) Find the zeros of $x^4 + 2x^3 - 7x^2 - 8x + 12$

Finding the Domain

Domain of Radical Expressions

The domain of $f(x) = \sqrt[n]{x}$ is:

- If n is even, set what's inside of the radical ≥ 0 and solve to x to get the domain
- If n id odd, the domain is ALWAYS $(-\infty, \infty)$

Ex) Find the domain. a) $\sqrt{4-x}$ b) $\sqrt[3]{x-6}$ c) $\sqrt[4]{3+x}$

Domain of Rational Expressions

Set the denominator equal to zero and solve for x. These are the values *not included* in the domain.

Ex) Find the domain. a) $\frac{x+2}{x^2+2x-15}$ b) $\frac{2x}{x^2-5x-6}$

Solving Trig Equations

1. Get all trig terms by themselves on one side of the equation.
2. Solve for the trig term by factoring, taking square roots, etc.
3. Evaluate your answer using the unit circle.

Ex) Solve. a) $2sin^2(x) - 1 = 0$ b) $cos^2(x) + 3\cos(x) + 2 = 0$

c) $2\cos(x) + \sqrt{2} = 0$

Verifying Trig Identities

Use reciprocal, quotient, and Pythagorean identities to manipulate one side of the equation to look like the other side.

Ex) Verify.

a) $\csc^2(x)\sec^2(x) = \csc^2(x) + \sec^2(x)$ b) $\frac{1}{1-\cos(x)} + \frac{1}{1+\cos(x)} = 2\csc^2(x)$

Approximating Decimal Radians as Degrees

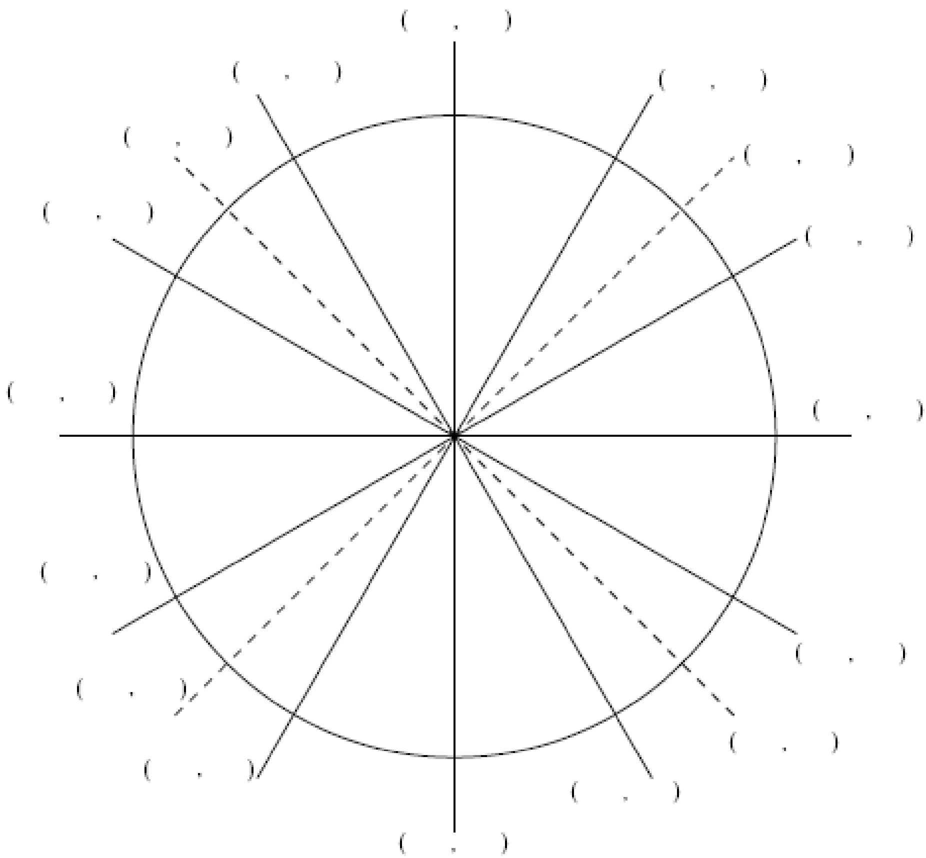
** Halfway around the unit circle is approximately 3.14 radians ($\pi \approx 3.14$ radians), which is 180°

Meaning ¼ of a rotation is approx. 1.57 radians, which is 90°, ¾ of a rotation is approx. 4.71 radians, which is 270° and one rotation is approximately 6.28 radians, which is 360°

Ex) Approximately how many degrees is 2 radians?

Ex) Approximately how many degrees is 3.5 radians?

Unit Circle Values



Formuals you will NEED to have MOMORIZED:

- Radian to degree conversion formula
- Degree to radian conversion formula
- $\boldsymbol{y = mx + b}$ ***OR*** $\boldsymbol{y - y_1 = m(x - x_1)}$
- Operations on functions
- Unit circle
- Reciprocal identities
- Quotient identities

You will be given this formula sheet for your final:

Exponents and logarithms	$a^x = b \quad \Leftrightarrow \quad x = \log_a b$
Laws of logarithms	$\log_c a + \log_c b = \log_c ab$ $\log_c a - \log_c b = \log_c \frac{a}{b}$ $\log_c a^r = r \log_c a$
Change of base	$\log_b a = \frac{\log_c a}{\log_c b}$

Length of an arc	$l = \theta r$
Area of a sector	$A = \frac{1}{2}\theta r^2$
Trigonometric identity	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
Double angle formulae	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Axis of symmetry of graph of a quadratic function	$f(x) = ax^2 + bx + c \quad \Rightarrow \quad$ axis of symmetry $x = -\frac{b}{2a}$
Relationships between logarithmic and exponential functions	$a^x = e^{x \ln a}$ $\log_a a^x = x = a^{\log_a x}$
Solutions of a quadratic equation	$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a \neq 0$
Discriminant	$\Delta = b^2 - 4ac$