Solve Nonlinear Inequalities

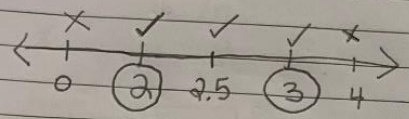
ex) $x^2 \leq 5x - 6$

$x^2 - 5x + 6 \leq 0$

$(x-3)(x-2) \leq 0$

$x = 3 \quad x = 2$

$\boxed{[2, 3]}$



$(x = 0)$

$0 - 0 + 6 \leq 0$

$6 \leq 0$ false

$(x = 2)$

$(2)^2 - 5(2) + 6 \leq 0$

$4 - 10 + 6 \leq 0$

$-6 + 6 \leq 0$

$0 \leq 0$ true

$(x = 2.5)$

$(2.5)^2 - 5(2.5) + 6 \leq 0$

$6.25 - 12.5 + 6 \leq 0$

$-.25 \leq 0$ true

$(x = 3)$

$(3)^2 - 5(3) + 6 \leq 0$

$9 - 15 + 6 \leq 0$

$-6 + 6 \leq 0$ true

$(x = 4)$

$(4)^2 - 5(4) + 6 \leq 0$

$16 - 20 + 6 \leq 0$

$-4 + 6 \leq 0$

$2 \leq 0$ false

ex) $3x^3 - 4x^2 - 12x > -16$

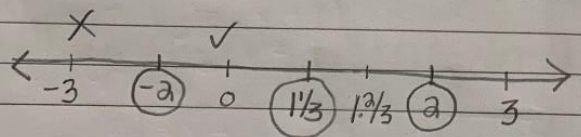
$(3x^3 - 4x^2 - 12x + 16) > 0$

$x^2(3x - 4) - 4(3x - 4) > 0$

$(x^2 - 4)(3x - 4) > 0$

$(x - 2)(x + 2)(3x - 4) > 0$

$x = 2 \quad x = -2 \quad x = 4/3 = 1\frac{1}{3}$



$(x = 0)$

$3(0)^3 - 4(0)^2 - 12(0) + 16 > 0$

$16 > 0$ true

$\boxed{(-2, 1\frac{1}{3})}$

$(x = -3)$

$3(-3)^3 - 4(-3)^2 - 12(-3) + 16 > 0$

$3(-27) - 4(9) + 36 + 16 > 0$

$-81 - 36 + 36 + 16 > 0$

$-65 > 0$ false

$(x = 2)$

$3(2)^3 - 4(2)^2 - 12(2) + 16 > 0$

$3(8) - 4(4) - 24 + 16 > 0$

$24 - 16 - 24 + 16 > 0$

$8 - 24 + 16 > 0$

$-16 + 16 > 0$

$0 > 0$

false

Solve exponential Equations

ex) $3^{x+2} = 18$

$\ln 3^{x+2} = \ln 18$

$(x+2)\ln 3 = \ln 18$

$x+2 = \frac{\ln 18}{\ln 3}$

$x = \frac{\ln 18}{\ln 3} - 2$

$x \approx .63$

ex) $5^{2x} = \frac{1}{5^x}$

$5^{2x} = 5^{-x}$

$2x = -x$

$3x = 0$

$x = 0$

ex) $1 + 6^{2x-1} = 10$

$6^{2x-1} = 9$

$\ln 6^{2x-1} = \ln 9$

$(2x-1)\ln 6 = \ln 9$

$2x - 1 = \frac{\ln 9}{\ln 6}$

$2x = (\ln 9 / \ln 6) + 1$

$x = \frac{(\ln 9 / \ln 6) + 1}{2}$

$x \approx 1.11$

Divide Polynomials

ex) use polyn. long division: $n^3 - 5n^2 - 7n + 2 \div n - 6$

$$\begin{array}{r} n^2 + n - 1 \\ n-6 \overline{)\, n^3 - 5n^2 - 7n + 2} \\ +(-n^3 + 6n^2) \\ \hline n^2 - 7n + 2 \\ +(-n^2 + 6n) \\ \hline -n + 2 \\ +(+n \cdot 6) \\ \hline -4 \end{array}$$

$\rightarrow n^2 + n - 1 - \frac{4}{n-6}$

ex) synthetic: $n^4 + n^3 - 4n - 4 \div n + 1$

$$\begin{array}{r|rrrrr} & 1 & 1 & 0 & -4 & -4 \\ -1 & \downarrow & -1 & 0 & 0 & 4 \\ \hline & 1 & 0 & 0 & -4 & 0 \end{array}$$

$n^3 - 4$

Write a Function Given its Zeros

ex) $5, -4, 1$

$(x-5)(x+4)(x-1)$

$(x-5)(x^2+3x-4)$

$x^3+3x^2-4x-5x^2-15x+20$

$\boxed{x^3-2x^2-19x+20}$

ex) $2, -2, 3$

$(x-2)(x+2)(x-3)$

$(x-2)(x^2-x-6)$

$x^3-x^2-6x-2x^2+2x+12$

$\boxed{x^3-3x^2-4x+12}$

Verify Trig Identities

ex) $\csc^2(x)\sec^2(x) = \csc^2(x)+\sec^2(x)$

$= \frac{1}{\sin^2(x)} + \frac{1}{\cos^2(x)}$

$= \frac{\cos^2(x)}{\sin^2(x)\cos^2(x)} + \frac{\sin^2(x)}{\sin^2(x)\cos^2(x)}$

$= \frac{\cos^2(x)+\sin^2(x)}{\sin^2(x)\cos^2(x)}$

$= \frac{1}{\sin^2(x)\cos^2(x)} = \frac{1}{\sin^2(x)} \cdot \frac{1}{\cos^2(x)}$

$= \csc^2(x)\sec^2(x)$

ex) $\frac{1}{1-\cos(x)} + \frac{1}{1+\cos(x)} = 2\csc^2(x)$

$\frac{1+\cos(x)}{(1-\cos(x))(1+\cos(x))} + \frac{1-\cos(x)}{(1-\cos(x))(1+\cos(x))}$

$\frac{1+\cos(x)+1-\cos(x)}{1-\cos^2(x)}$

$= \frac{2}{\sin^2(x)} = 2\csc^2(x)$

Solve Exponential Equations

ex) $3^{x+2} = 18$

$\ln 3^{x+2} = \ln 18$

$(x+2)\ln 3 = \ln 18$

$x+2 = \frac{\ln 18}{\ln 3}$

$x = \frac{\ln 18}{\ln 3} - 2$

$\boxed{x \approx .63}$

ex) $5^{2x} = \frac{1}{5^x}$

$5^{2x} = 5^{-x}$

$2x = -x$

$3x = 0$

$\boxed{x = 0}$

ex) $1 + 6^{2x-1} = 10$

$6^{2x-1} = 9$

$\ln 6^{2x-1} = \ln 9$

$(2x-1)\ln 6 = \ln 9$

$2x - 1 = \frac{\ln 9}{\ln 6}$

$2x = (\ln 9 / \ln 6) + 1$

$x = \frac{(\ln 9 / \ln 6) + 1}{2}$

$\boxed{x \approx 1.11}$

Divide Polynomials

ex) use polyn. long division: $n^3 - 5n^2 - 7n + 2 \div n - 6$

$$n-6 \overline{)\, n^3 - 5n^2 - 7n + 2}$$

$n^2 + n - 1$

$+(-n^3 + 6n^2)$

$n^2 - 7n + 2$

$+(-n^2 + 6n)$

$-n + 2$

$+(+n - 6)$

-4

$\rightarrow \boxed{n^2 + n - 1 - \frac{4}{n-6}}$

ex) synthetic: $n^4 + n^3 - 4n - 4 \div n + 1$

	1	1	0	-4	-4
-1	↓	-1	0	0	4
	1	0	0	-4	0

$\boxed{n^3 - 4}$

Graph log functions

ex) $y = -\log(x+5)+2$

$-y = \log(x+5)+2$

$-y-2 = \log(x+5)$

$10^{-y-2} = x+5$

$10^{-y-2} - 5 = x$

x	y
-4	-2
-5	-3

$y = -2$

$10^{-(-2)-2} - 5$

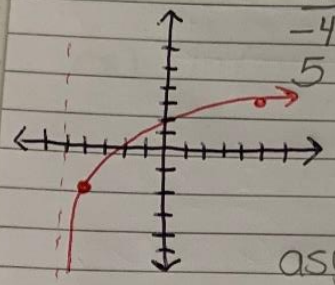
$10^{2-2} - 5$

$1 - 5 = -4$

$y = -3$

$10^{-(-3)-2} - 5$

$10^{3-2} - 5$



asym: $x = -5$

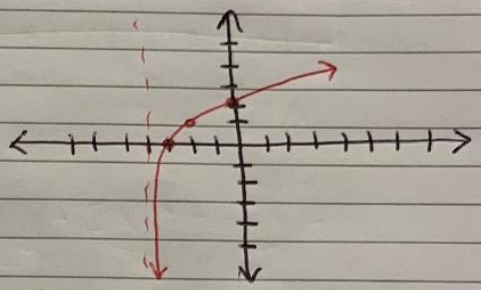
ex) $f(x) = \log_2(x+4)$

$y = \log_2(x+4)$

$2^y = x+4$

x	y
-3	0
-2	1
0	2

asym: $x = -4$



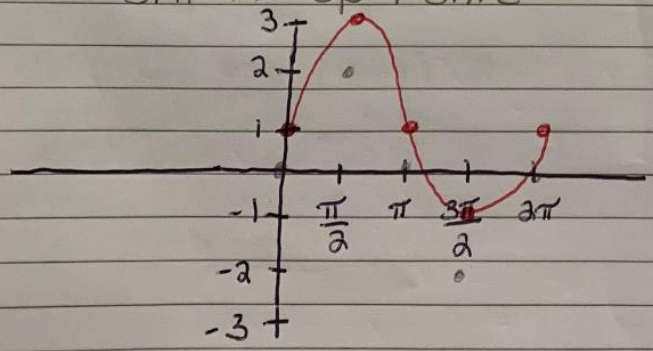
Graph Sine and Cosine Functions

a) $y = 2\sin(x)+1$

period: 2π

amp: 2

shift: up 1 unit

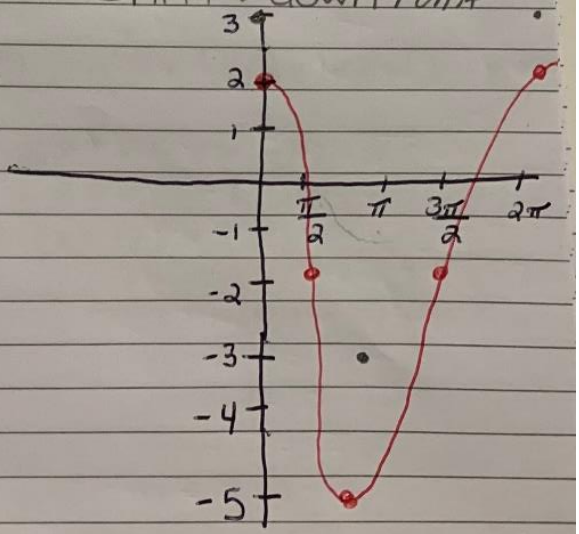


b) $y = 3\cos(x)-2$

period: 2π

Amp: 3

shift: down 1 unit



Operations on Functions

Ex) a.) $(f+g)(x)$
$3x+2+4-5x$
$\boxed{-2x+6}$

b.) $(f-g)(x)$
$3x+2-(4-5x)$
$3x+2-4+5x$
$\boxed{8x-2}$

c) $(fg)(x)$
$(3x+2)(4-5x)$
$12x-15x^2+8-10x$
$\boxed{-15x^2+2x+8}$

d.) $(f\circ g)(x) = f(g(x)) = 3(4-5x)+2 \rightarrow 12-15x+2 \rightarrow \boxed{-15x+14}$

e.) $(g\circ f)(x) = g(f(x)) = 4-5(3x+2) \rightarrow 4-15x-10 \rightarrow \boxed{-15x-6}$

f.) $f^{-1}(x) \Rightarrow y = 3x+2 \rightarrow x = 3y+2 \Rightarrow$
$x-2 = 3y$
$\frac{x-2}{3} = y$
$\boxed{f^{-1}(x) = \frac{x-2}{3}}$

g.) $(f\circ f^{-1})(x) \rightarrow f(f^{-1}(x)) \rightarrow 3\left(\frac{x-2}{3}\right)+2 = \frac{3x-6}{3}+2$
$\hookrightarrow \frac{3x-6}{3}+\frac{6}{3} = \frac{3x}{3} = \boxed{x}$

Solve Trig Equations

a) $2\sin^2(x)-1=0$
$\sin^2(x) = 1/2$
$\sin(x) = \pm\sqrt{1/2}$
$\sin(x) = \pm\sqrt{2}/2$

$\boxed{x = \frac{\pi}{4}}$
$\boxed{x = \frac{3\pi}{4}}$
$\boxed{x = \frac{5\pi}{4}}$
$\boxed{x = \frac{7\pi}{4}}$

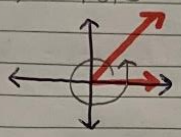
b) $\cos^2(x)+3\cos(x)+2=0$
$u = \cos(x)$
$u^2+3u+2=0$
$(u+2)(u+1)=0$
$(\cos(x)+2)(\cos(x)+1)=0$
$\cos(x)+2=0 \quad \cos(x)+1=0$
$\cos(x) = -2 \quad \cos(x) = -1$
$x =$ $\boxed{x = \pi}$

c) $2\cos(x)+\sqrt{2}=0$
$\cos(x) = -\sqrt{2}/2$

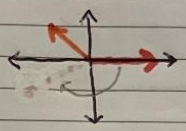
$\boxed{x = \frac{3\pi}{4}}$
$\boxed{x = \frac{5\pi}{4}}$

Sketch each in standard form

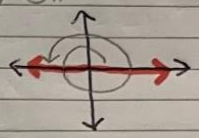
ex) 425°



ex) -210°



ex) 3π



Solve Natural Log Equations

ex) $\ln(3x+4) = \ln(4x+4)$

$3x+4 = 4x+4$

$4 = x+4$

$\boxed{0 = x}$

ex) $\log_3(x+2) - \log_3 \frac{1}{3} = 4$

$\log_3 \frac{x+2}{\frac{1}{3}} = 4$

$\log_3 \frac{x+2}{1} \cdot \frac{3}{1} = 4$

$\log_3 3(x+2) = 4$

$\log_3 3x+6 = 4$

$3^4 = 3x+6$

$\frac{3^4-6}{3} = x \rightarrow x = \frac{81-6}{3}$

$\boxed{x=25}$

↕ in wrong place

Solve Log Equations

ex) $6 + \log_5(x+1) = 8$

$\log_5(x+1) = 2$

$5^2 = x+1$

$25 = x+1$

$\boxed{24 = x}$

ex) $\ln(2x^2+12x) = \ln(-20+x^2)$

$2x^2+12x = -20+x^2$

$x^2+12x+20 = 0$

$(x+10)(x+2) = 0$

$\boxed{x=-10}$ $\boxed{x=-2}$

Alg 2 Trig 1B final study guide 2019 Key

Finding the inverse:

ex) $f(x) = -\frac{1}{3}x + 1$

$y = -\frac{1}{3}x + 1$

$x = -\frac{1}{3}y + 1$

$x - 1 = -\frac{1}{3}y$

$x - 1 = -\frac{y}{3}$

$3(x-1) = -y$

$3x - 3 = -y$

$-3x + 3 = y$

$f^{-1}(x) = -3x + 3$

ex) $y = 5 - \frac{2}{3}x$

$x = 5 - \frac{2}{3}y$

$x - 5 = -\frac{2y}{3}$

$3(x-5) = -2y$

$3x - 15 = -2y$

$-3x/2 + 15/2 = y$

$\frac{-3x+15}{2} = f^{-1}(x)$

Finding zeros

ex) $f(x) = \frac{8x^2 - 2}{-4 + 5x}$

$8x^2 - 2 = 0$

$8x^2 = 2$

$x^2 = 2/8$

$x^2 = 1/4$

$x = \pm 1/2$

ex) $y = 5x^2 + 18x - 8$

$-40 \quad 20 \quad -2$

$(5x^2 + 20x) - (2x - 8) = 0$

$5x(x+4) - 2(x+4) = 0$

$(5x-2)(x+4) = 0$

$x = 2/5$ $\quad x = -4$

Solve by completing the square

a.) $6x^2 + 12x - 18 = 0$

$6x^2 + 12x = 18$

$6(x^2 + 2x) = 18$

$(b/2)^2 = (2/2)^2 = 1^2 = 1$

$6(x^2 + 2x + 1) = 18 + 6$

$6(x+1)^2 = 24$

$(x+1)^2 = 4$

$x + 1 = \pm\sqrt{4}$

$x = \pm 2 - 1$

$x = 2 - 1$ $\quad x = -2 - 1$

$x = 1$ $\quad x = -3$

3
13

3 9
13 0

b) $m^2 + 18m - 88 = 0$

$m^2 + 18m = 88$

$(b/2)^2 = (18/2)^2 = 9^2 = 81$

$m^2 + 18m + 81 = 88 + 81$

$m^2 + 18m + 81 = 169$

$(m+9)^2 = 169$

$m + 9 = \pm\sqrt{169}$

$m + 9 = \pm 13$

$m = \pm 13 - 9$

$m = 13 - 9$ $\quad m = -13 - 9$

$m = 4$ $\quad m = -22$