Ch5 test Review 2019 Key

Find the exact value of each. Round to nearest thousandths place.

1.) $\cos 105°$

Use sum + difference formula:

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\cos(45°+60°) = \cos 45° \cos 60° - \sin 45° \sin 60°$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4} \approx \boxed{-.259}$

2.) $\cos(70°)\cos(25°) + \sin(70°)\sin(25°)$

Use sum + difference formula:

$\cos u \cos v + \sin u \sin v = \cos(u-v)$

$\cos 70° \cos 25° + \sin 70° \sin 25° = \cos(70°-25°)$

$= \cos(45°)$

$= \frac{\sqrt{2}}{2} \approx \boxed{.707}$

3.) Use power reducing formulas to write $\cos^2(x)\tan^2(x)$ in terms of a first power cosine

$\cos^2(x)\tan^2(x)$

$= \frac{1+\cos 2x}{2}\left(\frac{1-\cos 2x}{1+\cos 2x}\right)$

$= \frac{\cancel{(1+\cos 2x)}(1-\cos 2x)}{2\cancel{(1+\cos 2x)}}$

$= \frac{1-\cos 2x}{2}$

4.) Use product-to-sum formulas to write $\sin 6x \cos 3x$ as a sum or difference

$$\sin u \cos v = \frac{1}{2}[\sin(u+v)+\sin(u-v)]$$

$$\sin 6x \cos 3x = \frac{1}{2}[\sin(6x+3x)+\sin(6x-3x)]$$

$$= \frac{1}{2}[\sin(9x)+\sin(3x)]$$

OR

$$\frac{\sin(9x)+\sin(3x)}{2}$$

5.) Use sum-to-product formulas to find the exact value of $\cos 195° + \cos 105°$. Round to the nearest thousandth place.

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos 195° + \cos 105° = 2\cos\left(\frac{195+105}{2}\right)\cos\left(\frac{195-105}{2}\right)$$

$$= 2\cos\left(\frac{300}{2}\right)\cos\left(\frac{90}{2}\right)$$

$$= 2\cos(150°)\cos(45°)$$

$$= 2\left(\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{-2\sqrt{6}}{4} = \frac{-\sqrt{6}}{2} \approx \boxed{-1.225}$$

Solve each. #6-9 on $[0°, 360°)$, #10-13 on $0 \le x < 2\pi$

6.) $4\sin^2(x) + 7 = 10$

$4\sin^2(x) = 3$

$\sin^2(x) = \frac{3}{4}$

$\sin(x) = \pm\sqrt{\frac{3}{4}}$

$\sin(x) = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2}$

$\boxed{\begin{array}{l} x = 60° \\ x = 120° \\ x = 240° \\ x = 300° \end{array}}$

7.) $\cos(2x) + \cos(x) = 0$

$2\cos^2(x) - 1 + \cos(x) = 0$

$2\cos^2(x) + \cos(x) - 1 = 0$

use u substitution:

Let $u = \cos(x)$

$2u^2 + u - 1 = 0$

$(2u-1)(u+1) = 0$

$(2\cos(x)-1)(\cos(x)+1) = 0$

$2\cos(x) - 1 = 0 \quad \cos(x) + 1 = 0$

$2\cos(x) = 1 \quad \cos(x) = -1$

$\cos(x) = \frac{1}{2}$

$\boxed{\begin{array}{l} x = 60° \\ x = 300° \end{array}}$ $\boxed{x = 180°}$

8.) $\cot^2(x) + \csc(x) = 1$

$\csc^2(x) - 1 + \csc(x) = 1$

$\csc^2(x) - 1 + \csc(x) - 1 = 0$

$\csc^2(x) + \csc(x) - 2 = 0$

use u substitution

let $u = \csc(x)$

$u^2 + u - 2 = 0$

$(u+2)(u-1) = 0$

$(\csc(x)+2)(\csc(x)-1) = 0$

$\csc(x) + 2 = 0 \quad \csc(x) - 1 = 0$

$\csc(x) = -2 \quad \csc(x) = 1$

$\downarrow \quad \downarrow$

where $\sin(x) = -\frac{1}{2}$ where $\sin(x) = 1$

$\boxed{\begin{array}{l} x = 210° \\ x = 330° \end{array}}$ $\boxed{x = 90°}$

9.) $2\sin(2x) - \sqrt{3}\cos(x) = 0$

$2(2\sin(x)\cos(x)) - \sqrt{3}\cos(x) = 0$

$4\sin(x)\cos(x) - \sqrt{3}\cos(x) = 0$

$\cos(x)(4\sin(x) - \sqrt{3}) = 0$

$\cos(x) = 0 \quad 4\sin(x) - \sqrt{3} = 0$

$\boxed{\begin{array}{l} x = 270° \\ x = 90° \end{array}}$ $\cancel{\sin(x) = \frac{\sqrt{3}}{4}}$

$\cancel{x =}$

10.) $\tan^4(x) - 4\tan^2(x) + 3 = 0$

Use u substitution:

Let $u = \tan^2(x)$

$u^2 - 4u + 3 = 0$

$(u-3)(u-1) = 0$

$(\tan^2(x)-3)(\tan^2(x)-1) = 0$

$\tan^2(x) - 3 = 0 \quad \tan^2(x) - 1 = 0$

$\tan^2(x) = 3 \quad \tan^2(x) = 1$

$\tan(x) = \pm\sqrt{3} \quad \tan(x) = \pm\sqrt{1}$

$\tan(x) = \pm 1$

$\boxed{x = \frac{\pi}{3} \quad x = \frac{4\pi}{3} \quad x = \frac{2\pi}{3} \quad x = \frac{5\pi}{3}}$

$\boxed{x = \frac{\pi}{4} \quad x = \frac{5\pi}{4} \quad x = \frac{3\pi}{4} \quad x = \frac{7\pi}{4}}$

11.) $1 + \sin(x) = \cos^2(x)$

$1 + \sin(x) - \cos^2(x) = 0$

$1 + \sin(x) - (1 - \sin^2(x)) = 0$

$1 + \sin(x) - 1 + \sin^2(x) = 0$

$\sin^2(x) + \sin(x) = 0$

$\sin(x)(\sin(x) + 1) = 0$

$\sin(x) = 0 \quad \sin(x) + 1 = 0$

$\sin(x) = -1$

$\boxed{x = 0\pi \quad x = \pi} \quad \boxed{x = \frac{3\pi}{2}}$

12.) $\csc^2(x) - \sqrt{2}\csc(x) = 0$

$\csc(x)(\csc(x) - \sqrt{2}) = 0$

$\csc(x) = 0 \quad \csc(x) - \sqrt{2} = 0$

$\csc(x) = \sqrt{2}$

$\downarrow \quad \downarrow$

Where $\sin(x) = \frac{1}{0}$ undefined (nowhere)

where $\sin(x) = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin(x) = \frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4} \quad x = \frac{3\pi}{4}}$

13.) $2\cos^2(x) + \sin(x) = 1$

$2(1 - \sin^2(x)) + \sin(x) = 1$

$2 - 2\sin^2(x) + \sin(x) = 1$

$2 - 2\sin^2(x) + \sin(x) - 1 = 0$

$-2\sin^2(x) + \sin(x) + 1 = 0$

$-1(2\sin^2(x) - \sin(x) - 1) = 0$

-1 (Use u substitution:

Let $u = \sin(x)$

$-1(2u^2 - u - 1) = 0$

$-1(2u+1)(u-1) = 0$

$-1(2\sin(x)+1)(\sin(x)-1) = 0$

$\cancel{-1} \neq 0 \quad 2\sin(x) + 1 = 0 \quad \sin(x) - 1 = 0$

$\sin(x) = -1/2 \quad \sin(x) = 1$

$\boxed{x = \frac{7\pi}{6} \quad x = \frac{11\pi}{6}} \quad \boxed{x = \frac{\pi}{2}}$

Verify each.

14.) $\cos(x)+\sin(x)\tan(x)=\sec(x)$

$$\frac{\cos(x)}{1}+\frac{\sin(x)}{1}\cdot\frac{\sin(x)}{\cos(x)}=\sec(x)$$

$$\frac{\cos(x)}{1}+\frac{\sin^2(x)}{\cos(x)}=\sec(x)$$

$$\frac{\cos^2(x)}{\cos(x)}+\frac{\sin^2(x)}{\cos(x)}=\sec(x)$$

$$\frac{\cos^2(x)+\sin^2(x)}{\cos(x)}=\sec(x)$$

$$\frac{1}{\cos(x)}=\sec(x)\rightarrow \sec(x)=\sec(x)$$

15.) $\dfrac{\cos(x)}{1+\sin(x)}+\dfrac{\cos(x)}{1-\sin(x)}=2\sec(x)$

$$\frac{\cos(x)(1-\sin(x))}{(1+\sin(x))(1-\sin(x))}+\frac{\cos(x)(1+\sin(x))}{(1+\sin(x))(1-\sin(x))}=2\sec(x)$$

$$\frac{\cos(x)-\cos(x)\sin(x)}{(1+\sin(x))(1-\sin(x))}+\frac{\cos(x)+\cos(x)\sin(x)}{(1+\sin(x))(1-\sin(x))}=2\sec(x)$$

$$\frac{\cos(x)\cancel{-\cos(x)\sin(x)}+\cos(x)\cancel{+\cos(x)\sin(x)}}{(1+\sin(x))(1-\sin(x))}=2\sec(x)$$

$$\frac{2\cos(x)}{1-\sin(x)+\sin(x)-\sin^2(x)}=2\sec(x)\rightarrow\frac{2\cos(x)}{1-\sin^2(x)}=2\sec(x)\rightarrow\frac{2\cos(x)}{\cos^2(x)}=2\sec(x)\rightarrow\frac{2}{\cos(x)}=$$

$$2\sec(x)=2\sec(x)$$

16.) $\frac{1-\sin^2\theta}{\sin\theta(1+\sin\theta)}+1=\csc\theta$

$\frac{\cos^2\theta}{\sin\theta+\sin^2\theta}+1=\csc\theta$

$\frac{\cos^2\theta}{\sin\theta+\sin^2\theta}+\frac{\sin\theta+\sin^2\theta}{\sin\theta+\sin^2\theta}=\csc\theta$

$\frac{\cos^2\theta+\sin\theta+\sin^2\theta}{\sin\theta+\sin^2\theta}=\csc\theta$

$\frac{1+\cancel{\sin\theta}}{\sin\theta(1+\cancel{\sin\theta})}=\csc\theta$

$\frac{1}{\sin\theta}=\csc\theta \rightarrow \csc\theta=\csc\theta$

17.) $\tan^4(x)+2\tan^2(x)+1=\sec^4(x)$

$\tan^4(x)+2\tan^2(x)+1=(\sec^2(x))^2$

$\tan^4(x)+2\tan^2(x)+1=(1+\tan^2(x))^2$

$\tan^4(x)+2\tan^2(x)+1=(1+\tan^2(x))(1+\tan^2(x))$

$\tan^4(x)+2\tan^2(x)+1=1+\tan^2(x)+\tan^2(x)+\tan^4(x)$

$\tan^4(x)+2\tan^2(x)+1=1+2\tan^2(x)+\tan^4(x)$

$\tan^4(x)+2\tan^2(x)+1=\tan^4(x)+2\tan^2(x)+1$

18.) $\frac{\sin\theta}{\cos\theta+\sin\theta} + \frac{\sin\theta}{\cos\theta-\sin\theta} = \tan 2\theta$

$$\frac{\sin\theta(\cos\theta-\sin\theta)}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)} + \frac{\sin\theta(\cos\theta+\sin\theta)}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)} = \tan 2\theta$$

$$\frac{\sin\theta\cos\theta - \cancel{\sin^2\theta} + \sin\theta\cos\theta + \cancel{\sin^2\theta}}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)} = \tan 2\theta$$

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \cancel{\cos\theta\sin\theta} + \cancel{\cos\theta\sin\theta} - \sin^2\theta} = \tan 2\theta$$

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \tan 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$$

$$\tan 2\theta = \tan 2\theta$$

19.) $\csc^2(x)\sec^2(x) = \csc^2(x) + \sec^2(x)$

$$\csc^2(x)\sec^2(x) = \frac{1}{\sin^2(x)} + \frac{1}{\cos^2(x)}$$

$$\csc^2(x)\sec^2(x) = \frac{\cos^2(x)}{\sin^2(x)\cos^2(x)} + \frac{\sin^2(x)}{\sin^2(x)\cos^2(x)}$$

$$\csc^2(x)\sec^2(x) = \frac{\cos^2(x)+\sin^2(x)}{\sin^2(x)\cos^2(x)}$$

$$\csc^2(x)\sec^2(x) = \frac{1}{\sin^2(x)\cos^2(x)} = \frac{1}{\sin^2(x)} \cdot \frac{1}{\cos^2(x)}$$

$$\csc^2(x)\sec^2(x) = \csc^2(x)\sec^2(x)$$