A3: Polynomials and Factoring

Polynomials

__Polynomial:__ an expression with *more than one term* (addition and/or subtraction must occur at least one time)
Examples) $17x + 4$ $14x^2 + 2x - 1$ $6y^7 - 5y^6 + 2y^5 - y^4 + 2y^2 + 1$

Remember, a ***monomial is just one single term, with only multiplication OR one single division bar Ex) $-52a^3bc^{17}$

__Standard form of a polynomial__ is when it is written in *descending powers* of the variable. This is the proper format. When a polynomial has more than one variable, it's standard form is written with variable terms in alphabetic order, as well as in descending order by the sums of the exponents for each term.

Examples of polynomials in standard form:

$14x^7 + 6x^5 - 4x^4 + 3x^2 - 10$ $4x^3 + 3xy - y^2$ $x^5y^2 + 2xy^2 - 4xy + 11$

__The leading coefficient of the polynomial__ is the coefficient on the very first term when the polynomial is in standard form.

__The degree of a polynomial:__ when there is *only one variable*, it is the exponent of the variable of the very first term when the polynomial is in standard form.
If there is *more than one variable*, it is the greatest sum of the exponents of the terms.

Examples: $-14x^2 + 6x + 1$ degree is 2 $2x^3y^{10} + 6xy^8 - 2x^6y^9$ degree is 15

Adding and Subtracting polynomials

__Like terms__ are monomials that have the same variables, and those variables have the same exponents.
When we combine them (by adding or subtracting), we only perform the operation on the coefficients, while the variable and it's exponent stays the same.

Examples) $6x^2 + 12x^2 = 18x^2$ $6y^3 - 6y^3$ =cannot be simplified, different exponents on y

__Adding polynomials:__ Drop parentheses, and combine like terms

a) Find the sum

__Subtracting polynomials:__ Rewrite the polynomial in the 1st set on parentheses as is. Then, distribute the negative sign to all of the terms in the second polynomial (this changes the sign of each term to its opposite), and write each "new term" from the 2nd polynomial next to the 1st, and combine like terms.

b) Find the difference

Multiplying Polynomials

Multiply each term in the given polynomials by each term in the other polynomial(s). Then, combine like terms.

Find each product a) b)

Some binomial products have special formulas that can be used to multiply. You do not have to memorize these, and can simply apply the distributive property. However, these can be helpful shortcuts:

SPECIAL PRODUCT PATTERNS		EXAMPLES
SUM & DIFFERENCE	$(a+b)(a-b) =$ $a^2 - b^2$	$(x+3)(x-3) =$ $x^2 - 9$
SQUARE OF A BINOMIAL	$(a+b)^2 =$ $a^2 + 2ab + b^2$	$(y+4)^2 =$ $y^2 + 8y + 16$
	$(a-b)^2 =$ $a^2 - 2ab + b^2$	$(3t^2 - 2)^2 =$ $9t^4 - 12t^2 + 4$
CUBE OF A BINOMIAL	$(a+b)^3 =$ $a^3 + 3a^2b + 3ab^2 + b^3$	$(x+1)^3 =$ $x^3 + 3x^2 + 3x + 1$
	$(a-b)^3 =$ $a^3 - 3a^2b + 3ab^2 - b^3$	$(p-2)^3 =$ $p^3 - 6p^2 + 12p - 8$

Find each product. c) d)

Factoring Polynomials

<u>Factoring a polynomial</u> means to *write the polynomial as a product*. There are many different ways and forms to factor. Often, you may need to factor a polynomial more than once, using more than one method of factoring.

Factor the Difference of Two Squares $\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$

e)

f)

Factor by taking out the greatest common factor

g)

Factor a Perfect Square Trinomial $\boldsymbol{a^2 + 2ab + b^2 = (a + b)^2}$ $\boldsymbol{a^2 - 2ab + b^2 = (a - b)^2}$

h)

i)

Factor the difference of Cubes $\boldsymbol{a^3 - b^3 = (a + b)(a^2 - ab + b^2)}$

j.)

k.)

Factor the Sum of Cubes $\boldsymbol{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$

l.)

m.)

Factor a Trinomial: Leading Coefficient is 1

Find 2 numbers that you can add to get the coefficient of the middle term, that also can by multiplied to get the constant (the last term). These numbers are the second part of each set of parentheses, set up like this (x)(x), and the sign of each number is the sign that is in between itself and the variable.

n.)

o.)

Factor a 4 Piece Polynomial – Factor by Grouping

1.) Create parentheses around the first 2 terms, and the last 2 terms, creating 2 binomials. The sign in between the 2 sets of parentheses should not be included in either binomial
2.) Factor out a GCF of each binomial, if possible.
3.) What is left inside of each set of parentheses should be exactly the same, always.
4.) Write your answer as a product, using 2 sets of parentheses. One set will contain the binomial that should appear twice now in your newly written polynomial (we do not account for this binomial twice in the solution), and the second set will contain the terms that were factored out of each binomial in step 2, along with the signs attached.

p.)

q.)

Factor a Trinomial: Leading Coefficient is NOT 1

1.) Multiply the first and last terms (coefficient on the first term × the constant at the end)
2.) Find a pair pf numbers whose sum is the coefficient of the middle term, and whose product is the same as the one found in step one
3.) Rewrite your polynomial – it will now become a 4 piece polynomial rather than 3. The middle term will be rewritten as the sum of the 2 numbers found in step 2, along with the variable (these will be like terms. If you simplify, you should end up with the trinomial you started with).
4.) Factor by grouping

r.)

s.)