



Mathematics SL formula booklet

For use during the course and in the examinations

First examinations 2014

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Prior learning

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| Area of a parallelogram | $A = b \times h$ |
| Area of a triangle | $A = \frac{1}{2}(b \times h)$ |
| Area of a trapezium | $A = \frac{1}{2}(a + b)h$ |
| Area of a circle | $A = \pi r^2$ |
| Circumference of a circle | $C = 2\pi r$ |
| Volume of a pyramid | $V = \frac{1}{3}(\text{area of base} \times \text{vertical height})$ |
| Volume of a cuboid (rectangular prism) | $V = l \times w \times h$ |
| Volume of a cylinder | $V = \pi r^2 h$ |
| Area of the curved surface of a cylinder | $A = 2\pi r h$ |
| Volume of a sphere | $V = \frac{4}{3}\pi r^3$ |
| Volume of a cone | $V = \frac{1}{3}\pi r^2 h$ |
| Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) | $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ |
| Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2) | $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$ |

Topic I—Algebra

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| <p>1.1</p> | <p>The n^{th} term of an arithmetic sequence</p> <p>The sum of n terms of an arithmetic sequence</p> <p>The n^{th} term of a geometric sequence</p> <p>The sum of n terms of a finite geometric sequence</p> <p>The sum of an infinite geometric sequence</p> | $u_n = u_1 + (n-1)d$ $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$ $u_n = u_1 r^{n-1}$ $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$ $S_\infty = \frac{u_1}{1 - r}, \quad r < 1$ |
| <p>1.2</p> | <p>Exponents and logarithms</p> <p>Laws of logarithms</p> <p>Change of base</p> | $a^x = b \Leftrightarrow x = \log_a b$ $\log_c a + \log_c b = \log_c ab$ $\log_c a - \log_c b = \log_c \frac{a}{b}$ $\log_c a^r = r \log_c a$ $\log_b a = \frac{\log_c a}{\log_c b}$ |
| <p>1.3</p> | <p>Binomial coefficient</p> <p>Binomial theorem</p> | $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$ |

Topic 2—Functions and equations

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| 2.4 | Axis of symmetry of graph of a quadratic function | $f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry $x = -\frac{b}{2a}$ |
| 2.6 | Relationships between logarithmic and exponential functions | $a^x = e^{x \ln a}$ $\log_a a^x = x = a^{\log_a x}$ |
| 2.7 | Solutions of a quadratic equation Discriminant | $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$ $\Delta = b^2 - 4ac$ |

Topic 3—Circular functions and trigonometry

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| 3.1 | Length of an arc Area of a sector | $l = \theta r$ $A = \frac{1}{2} \theta r^2$ |
| 3.2 | Trigonometric identity | $\tan \theta = \frac{\sin \theta}{\cos \theta}$ |
| 3.3 | Pythagorean identity Double angle formulae | $\cos^2 \theta + \sin^2 \theta = 1$ $\sin 2\theta = 2 \sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$ |
| 3.6 | Cosine rule Sine rule Area of a triangle | $c^2 = a^2 + b^2 - 2ab \cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $A = \frac{1}{2} ab \sin C$ |

Topic 4—Vectors

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| 4.1 | Magnitude of a vector | $ \mathbf{v} = \sqrt{v_1^2 + v_2^2 + v_3^2}$ |
| 4.2 | Scalar product | $\mathbf{v} \cdot \mathbf{w} = \mathbf{v} \mathbf{w} \cos\theta$ $\mathbf{v} \cdot \mathbf{w} = v_1w_1 + v_2w_2 + v_3w_3$ |
| | Angle between two vectors | $\cos\theta = \frac{\mathbf{v} \cdot \mathbf{w}}{ \mathbf{v} \mathbf{w} }$ |
| 4.3 | Vector equation of a line | $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ |

Topic 5—Statistics and probability

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| 5.2 | Mean of a set of data | $\bar{x} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$ |
| 5.5 | Probability of an event A | $P(A) = \frac{n(A)}{n(U)}$ |
| | Complementary events | $P(A) + P(A') = 1$ |
| 5.6 | Combined events | $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ |
| | Mutually exclusive events | $P(A \cup B) = P(A) + P(B)$ |
| | Conditional probability | $P(A \cap B) = P(A)P(B A)$ |
| | Independent events | $P(A \cap B) = P(A)P(B)$ |
| 5.7 | Expected value of a discrete random variable X | $E(X) = \mu = \sum_x x P(X = x)$ |
| 5.8 | Binomial distribution | $X \sim B(n, p) \Rightarrow P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}, r = 0, 1, \dots, n$ |
| | Mean | $E(X) = np$ |
| | Variance | $\text{Var}(X) = np(1-p)$ |
| 5.9 | Standardized normal variable | $z = \frac{x - \mu}{\sigma}$ |

Topic 6—Calculus

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| 6.1 | Derivative of $f(x)$ | $y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$ |
| 6.2 | Derivative of x^n Derivative of $\sin x$ Derivative of $\cos x$ Derivative of $\tan x$ Derivative of e^x Derivative of $\ln x$ Chain rule Product rule Quotient rule | $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$ $f(x) = \sin x \Rightarrow f'(x) = \cos x$ $f(x) = \cos x \Rightarrow f'(x) = -\sin x$ $f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$ $f(x) = e^x \Rightarrow f'(x) = e^x$ $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$ $y = g(u), u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ |
| 6.4 | Standard integrals | $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$ $\int \frac{1}{x} dx = \ln x + C, \quad x > 0$ $\int \sin x dx = -\cos x + C$ $\int \cos x dx = \sin x + C$ $\int e^x dx = e^x + C$ |
| 6.5 | Area under a curve between $x = a$ and $x = b$ Volume of revolution about the x -axis from $x = a$ to $x = b$ | $A = \int_a^b y dx$ $V = \int_a^b \pi y^2 dx$ |
| 6.6 | Total distance travelled from t_1 to t_2 | $\text{distance} = \int_{t_1}^{t_2} v(t) dt$ |

