Algebra 2 Trig IB Final Exam Outline & Review 2019

Finding the Inverse of a Function

1. Replace "f(x)" with "y" unless the equation already involves "y"
2. Switch x and y
3. Solve the equation for y
4. Replace "y" with $f^{-1}(x)$ and that is your inverse

Ex) Find the inverse of $f(x) = -\frac{1}{3}x + 1$

Ex) Find the inverse of $y = 5 - \frac{2}{3}x$

Finding zeros of a function

1. Set = to 0 and solve (factor the left side, if it's a rational function, set the numerator = to zero and solve)

Ex) Find the zeros of the function $f(x) = \frac{8x^2-2}{-4+5x}$

Ex) Find the zeros of the function $y = 5x^2 + 18x - 8$

Solve a quadratic by completing the square

1. Add/subtract the constant over to the right side of the = sign
2. If the leading coefficient is NOT 1, factor it out of the left side (if not, skip to step 3)
3. Find $\left(\frac{b}{2}\right)^2$ and add it to BOTH sides of the equation
4. Factor the left side of the equation (should factor into a binomial squared)
5. Solve the equation

Ex) Solve by completing the square a.) $6x^2 + 12x - 18 = 0$ b.) $m^2 + 18m - 88 = 0$

Graph log functions

Find the domain, range, x/y intercepts, and vertical asymptotes and apply them to the graph (see notes from section 3.2 (part 2) for more details)

Ex) Graph $y = -\log_2(x+5) + 2$

Ex) Graph $f(x) = \log_2(x+4)$

Graph Sine/Cosine Functions

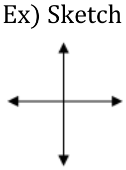
It is easiest to graph the parent function of sine/cosine first. Then, apply the translations included in the given function to create it's graph. (see notes from 4.5 for these details)

Ex) Graph at least one full period of: a) $y = 2\sin(x) + 1$ b) $y = 3\cos(x) - 2$

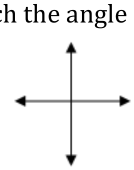
Sketch an angle in standard position

1. Create the initial side of the angle and make sure it is bold and visible on the graph
2. Visualize rotating the terminal side of the angle (counterclockwise for a positive angle, and clockwise for a negative angle) beginning at the initial side, adding 360° (2π) with each full rotation
3. Draw an arrow illustrating how many times the terminal side rotates around the circle, in what direction, and where it stops

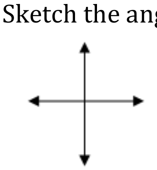
Ex) Sketch the angle 425°



Ex) Sketch the angle $-210°$



Ex) Sketch the angle 3π



Solve Natural log Equations

1. Isolate the "ln(x)" part of the equation on one side of the equal sign by moving everything else over
2. Write each side of the equation as an exponent on the natural exponential e
3. On the side of the equation in which the ln(x) was placed as an exponent on the natural exponential e, the "ln" and "e" will cancel out (nothing will cancel out on the other side of the equals sign)
4. Solve the equation using a calculator

Ex) Solve $\ln(3x+4) = \ln(4x+4)$

Ex) Solve $\ln(2x^2+12x) = \ln(-20+x^2)$

Solving Logarithmic Equations

1. Isolate the logarithm part of the equation (this may involve condensing using properties of logs if there is more than one log term
2. Rewrite the log using the property $\log_a x \rightarrow a^{\log_a x}$ to get the variable out of the log
3. Solve the remaining equation for the variable

Ex) Solve $6 + \log_5(x+1) = 8$

Ex) Solve $\log_3(x+2) - \log_3 \frac{1}{3} = 4$

Solving Non-Linear Inequalities

1. Move all terms to be on one side of the inequality sign, and on the other side is only 0
2. Factor the side with the polynomial, and set each factor = to zero and solve. These values are your critical numbers
3. Make a number line and mark all critical numbers in numerical order
4. Pick one number that falls before and after each critical number
5. Plug the numbers from step 4 into the original inequality. If you get a true statement, solutions to the inequality fall on this interval, and if it is a false statement, solutions for not fall on this interval
6. If the inequality sign is less than or equal to OR greater than or equal to, be sure to plug in the critical points included in the intervals that make up your answer

Ex) Solve $x^2 \leq 5x - 6$

Ex) Solve $3x^3 - 4x^2 - 12x > -16$

Solving Exponential Equations

1. Isolate the exponential part of the equation (the part that has a variable in the exponent)
2. Rewrite the equation by "taking the ln of both sides" using the property $a^x \rightarrow \ln(a^x) \rightarrow x \ln a$ to get the variable out of the exponent
3. Solve the remaining equation for the variable, using your calculator to evaluate any ln terms

Ex) Solve $3^{x+2} = 18$

Ex) $5^{2x} = \frac{1}{5^x}$

Ex) $1 + 6^{2x-1} = 10$

Divide polynomials using polynomial long division or synthetic division

Polynomial long division:

Ex) Divide $6x^3 - 19x^2 + 16x - 4$ *by* $x - 2$ and use the result to factor the polynomial completely.

Solution

$$
\begin{array}{r l}
 & 6x^2 - 7x + 2 \\
x - 2 \,\big)\!\! & 6x^3 - 19x^2 + 16x - 4 \\
 & \underline{6x^3 - 12x^2} \\
 & \quad -7x^2 + 16x \\
 & \quad \underline{-7x^2 + 14x} \\
 & \qquad\quad 2x - 4 \\
 & \qquad\quad \underline{2x - 4} \\
 & \qquad\qquad 0
\end{array}
$$

Think $\frac{6x^3}{x} = 6x^2$.
Think $\frac{-7x^2}{x} = -7x$.
Think $\frac{2x}{x} = 2$.

Multiply: $6x^2(x - 2)$.
Subtract.
Multiply: $-7x(x - 2)$.
Subtract.
Multiply: $2(x - 2)$.
Subtract.

From this division, you can conclude that

$6x^3 - 19x^2 + 16x - 4 = (x - 2)(6x^2 - 7x + 2)$

and by factoring the quadratic $6x^2 - 7x + 2$, you have

$6x^3 - 19x^2 + 16x - 4 = (x - 2)(2x - 1)(3x - 2)$.

x-2=0 → x=2   2x-1=0 → x= ½   3x-2=0 → x=2/3

Synthetic Division:

<table>
<tr><th>Problem</th><th>Solution</th></tr>
<tr><td>$(5x^3+6x^2-45x-54)$
$\div(x+3)$</td><td>Use synthetic division, using the first root, which is -3:
$\begin{array}{r|rrrr} -3 & 5 & 6 & -45 & -54 \\ & & -15 & 27 & 54 \\ \hline & 5 & -9 & -18 & \boxed{0} \end{array}$
Now we end up with $5x^2-9x-18$. Factor (unfoil) to get $(5x+6)(x-3)$.
The factors are $(x+3)$, $(5x+6)$, and $(x-3)$, and the real roots are -3, $-\frac{6}{5}$, and 3.</td></tr>
<tr><td>$(3x^4-24x)$
$\div(x-2)$</td><td>First, take GCF 3x out of the top (dividend) to get x^3-8. Then use synthetic division, using the first root, which is 2. Notice that we have to use 0's for the x^2 and x terms:
$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & \boxed{0} \end{array}$
Now we end up with x^2+2x+4, which we can't factor (it has a negative discriminant). So this factor has no real roots.
The factors are $(3)(x)(x-2)$, and (x^2+2x+4), and the real roots are 0, and 2 (we don't need to worry about the 3). Look familiar? It makes sense that the root of x^3-8 is 2; since 2 is the cube root of 8.</td></tr>
<tr><td>$(10x^4-13x^3-21x^2+10x+8)$
$\div(x-2)$</td><td>Use synthetic division, using the first root, which is 2:
$\begin{array}{r|rrrrr} 2 & 10 & -13 & -21 & 10 & 8 \\ & & 20 & 14 & -14 & -8 \\ \hline -1 & 10 & 7 & -7 & -4 & \boxed{0} \\ & & -10 & 3 & 4 & \\ \hline & 10 & -3 & -4 & \boxed{0} & \end{array}$
Since we ended up with 4 terms, we can use synthetic division again, but we have to guess on a factor. If you don't get remainder of 0, we have to guess again. -1 works, and then we get $10x^2-3x-4$; factor to get $(5x-4)(2x+1)$.
The factors are $(x-2)$, $(x+1)$, $(5x-4)$, and $(2x+1)$, and the real roots are -1, $-\frac{1}{2}$, $\frac{4}{5}$, and 2.</td></tr>
</table>

Ex) Divide the polynomials using polynomial long division: $(n^3 - 5n^2 - 7n + 2) \div (n - 6)$

Ex) Divide the polynomials using synthetic division: $(n^4 + n^3 - 4n - 4) \div (n + 1)$

Writing a Function Given the Zeros

Ex) Write a function with the given the zeros. -1, 1, 6

If a polynomial has zeros, then you can write:

- $x = -1 \quad (x + 1)$
- $x = 1 \quad (x - 1)$
- $x = 6 \quad (x - 6)$

So, we have $(x + 1)(x - 1)(x - 6) = x^3 - 6x^2 - x + 6$

Ex) Write a function with the given the zeros.

a) $5, -4, 1$

b) $2, -2, 3$

Verifying Trig Identities

Use reciprocal, quotient, and Pythagorean identities to manipulate one side of the equation to look like the other side.

Ex) Verify. a) $\csc^2(x)\sec^2(x) = \csc^2(x) + \sec^2(x)$

b) $\frac{1}{1-\cos(x)} + \frac{1}{1+\cos(x)} = 2\csc^2(x)$

Operations on Functions

$(f+g)(x) = f(x) + g(x)$ $(f-g)(x) = f(x) - g(x)$ $(fg)(x) = f(x) \bullet g(x)$

$(f \circ g)(x) = f(g(x))$ $(g \circ f)(x) = g(f(x)$ $(f \circ f^{-1})(x) = f(f^{-1}(x))$

Ex) Given $f(x) = 3x + 2$ and $g(x) = 4 - 5x$ find the following.

a) $(f+g)(x)$ b) $(f-g)(x)$ c) $(fg)(x)$
a) $(f \circ g)(x)$ e) $(g \circ f)(x)$ f) $f^{-1}(x)$

g) $(f \circ f^{-1})(x)$

Solving Trig Equations

1. Get all trig terms by themselves on one side of the equation.
2. Solve for the trig term by factoring, taking square roots, etc.
3. Evaluate your answer using the unit circle.

Ex) Solve. a) $2sin^2(x) - 1 = 0$ b) $cos^2(x) + 3\cos(x) + 2 = 0$ c) $2\cos(x) + \sqrt{2} = 0$