Solving logarithmic and exponential expressions

THE GOAL IN SOLVING ALL OF THESE EQUATIONS IS TO SOLVE FOR "x" BY GETTING "x" BY ITSELF

These inverse formulas will help us to get x by itself when "x" is included in an ln, log, or e

Inverse Property	When do I use this?	How do I use this?
$$e^{\ln x} = x$$ Natural log → exponential $\ln x \quad \rightarrow \quad e^x$	We use this when the part of the equation that has the "x" is a part of a natural log $\ln x$	**Example: Solve $\mathbf{\ln(3x) = 18}$** 1. The "x" term is found in the natural log, so, we can rewrite it as $e^{\ln(3x)} = e^{18}$ *(What we do to one side we have to do to the other)* 2. The e and the ln act in such a way that they cancel each other out, so we are left with $3x = e^{18}$ 3. Continue to solve for x $\frac{3x}{3} = \frac{e^{18}}{3} \quad \rightarrow x = \frac{e^{18}}{3}$
$$\ln e^x = x$$ Exponential → natural log $e^x \quad \rightarrow \quad \ln x$	When the part of the equation that has the "x" is written as the exponent of e	**Example: Solve $\mathbf{2e^x = 12}$** 1. We want to get as close as we can to getting "x" by itself, so we can divide each side by 2 $\frac{2e^x}{2} = \frac{12}{2} \rightarrow e^x = 6$ 2. The "x" term is found as an exponent of e, so we can "take the ln of both sides" and rewrite as: $\ln e^x = \ln 6$ 3. The e and the ln act in such a way that they cancel each other out, so we are left with $x = \ln 6$
$$a^{\log_a x} = x$$ Logarithm → exponential with base a (not base e) $\log_a x \quad \rightarrow \quad a^{\log_a x}$	When the part of the equation that has the "x" is a part of a logarithm $\log_a x$	**Example: Solve $\mathbf{\log_5(x+1) = 2}$** 1. The "x" term is found within a log, so we can rewrite this as $5^{\log_5(x+1)} = 5^2$ → *notice how we used the same base to rewrite each side* 2. Because we wrote this log as an exponent, we are left with $(x+1) = 5^2$ 3. Continue to solve for x: $(x+1) = 5^2$ $x + 1 = 25$ $-1 \quad -1$ $x = 24$

What if the “x” is the power of a whole number instead of e?

Ex) Solve $\mathbf{5^x = 7}$

1. *“Take the ln” of both sides*
   $\ln 5^x = \ln 7$
2. Apply the power property by bringing the exponent to the front (this is to get the “x” out of being in the position of an exponent so it is easier to solve for)
   $x \ln 5 = \ln 7$

3. Continue to solve for x (getting x alone on one side)
   Divide both sides by $\ln 5$ → $\frac{x \ln 5}{\ln 5} = \frac{\ln 7}{\ln 5}$

   And we have $x = \frac{\ln 7}{\ln 5}$

   ** When you are solving a problem like this, where “x” is the exponent of a whole number rather than e, “taking the ln” of both sides allows you to apply the power property which gets the “x” out of the exponent position making it possible to solve for “x” by hand

THE PRODUCT, QUOTIENT, AND POWER PROPERTIES CANNOT BE APPLIED UNLESS WHAT WE ARE WORKING WITH IS IN TERMS OF $\ln x$ OR $\log_a x$