5-2 Study Guide and Intervention

Solving Inequalities by Multiplication and Division

Solve Inequalities by Multiplication If each side of an inequality is multiplied by the same positive number, the resulting inequality is also true. However, if each side of an inequality is multiplied by the same negative number, the direction of the inequality must be reversed for the resulting inequality to be true.

Multiplication Property of Inequalities	For all numbers a, b, and c, with $c \neq 0$, **1.** if c is positive and $a > b$, then $ac > bc$; if c is positive and $a < b$, then $ac < bc$; **2.** if c is negative and $a > b$, then $ac < bc$; if c is negative and $a < b$, then $ac > bc$.

The property is also true when $>$ and $<$ are replaced with $\geq$ and $\leq$.

Example 1 **Solve $-\frac{y}{8} \leq 12$.**

$-\frac{y}{8} \geq 12$ Original inequality

$(-8)\left(-\frac{y}{8}\right) \leq (-8)12$ Multiply each side by -8; change $\geq$ to $\leq$.

$y \leq -96$ Simplify.

The solution is $\{y \mid y \leq -96\}$.

Example 2 **Solve $\frac{3}{4}k < 15$.**

$\frac{3}{4}k < 15$ Original inequality

$\left(\frac{4}{3}\right)\frac{3}{4}k < \left(\frac{4}{3}\right)15$ Multiply each side by $\frac{4}{3}$.

$k < 20$ Simplify.

The solution is $\{k \mid k < 20\}$.

Exercises

Solve each inequality. Check your solution.

1. $\frac{y}{6} \leq 2$

2. $-\frac{n}{50} > 22$

3. $\frac{3}{5}h \geq -3$

4. $-\frac{p}{6} < -6$

5. $\frac{1}{4}n \geq 10$

6. $-\frac{2}{3}b < \frac{1}{3}$

7. $\frac{3m}{5} < -\frac{3}{20}$

8. $-2.51 \leq -\frac{2h}{4}$

9. $\frac{g}{5} \geq -2$

10. $-\frac{3}{4} > -\frac{9p}{5}$

11. $\frac{n}{10} \geq 5.4$

12. $\frac{2a}{7} \geq -6$

Define a variable, write an inequality, and solve each problem. Check your solution.

13. Half of a number is at least 14.

14. The opposite of one-third a number is greater than 9.

15. One fifth of a number is at most 30.

5-2 Study Guide and Intervention *(continued)*

Solving Inequalities by Multiplication and Division

Solve Inequalities by Division If each side of a true inequality is divided by the same positive number, the resulting inequality is also true. However, if each side of an inequality is divided by the same negative number, the direction of the inequality symbol must be reversed for the resulting inequality to be true.

Division Property of Inequalities	For all numbers a, b, and c with $c \neq 0$, **1.** if c is positive and $a > b$, then $\frac{a}{c} > \frac{b}{c}$; if c is positive and $a < b$, then $\frac{a}{c} < \frac{b}{c}$; **2.** if c is negative and $a > b$, then $\frac{a}{c} < \frac{b}{c}$; if c is negative and $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

The property is also true when $>$ and $<$ are replaced with $\geq$ and $\leq$.

Example **Solve $-12y \geq 48$.**

$-12y \geq 48$ Original inequality

$\frac{-12y}{-12} \leq \frac{48}{-12}$ Divide each side by -12 and change $\geq$ to $\leq$.

$y \leq -4$ Simplify.

The solution is $\{y \mid y \leq -4\}$.

Exercises

Solve each inequality. Check your solution.

1. $25g \geq -100$

2. $-2x \geq 9$

3. $-5c > 2$

4. $-8m < -64$

5. $-6k < \frac{1}{5}$

6. $18 < -3b$

7. $30 < -3n$

8. $-0.24 < 0.6w$

9. $25 \geq -2m$

10. $-30 > -5p$

11. $-2n \geq 6.2$

12. $35 < 0.05h$

13. $-40 > 10h$

14. $-\frac{2}{3n} \geq 6$

15. $-3 < \frac{p}{4}$

16. $4 > \frac{-x}{2}$

Define a variable, write an inequality, and solve each problem. Then check your solution.

17. Four times a number is no more than 108.

18. The opposite of three times a number is greater than 12.

19. Negative five times a number is at most 100.