Sequences and Series Formula Sheet

Section 1:

- **Factorial**: If n is a positive integer, n factorial is defined as

$$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \cdot \cdot (n-1) \cdot n.$$

- **Summation Notation** (or Sigma Notation): The notation used to represent the sum of the terms of a finite sequence. $\sum_{i=1}^{n} a_1 = a_1 + a_2 + a_3 + a_4 + \cdots a_n$

  where i is called **the index of summation**, n is the **upper limit of summation**, and 1 is the lower **limit of summation**.

- **Series**: The sum of the terms of a finite or infinite sequence.

  - The sum of the first n terms of the sequence is called a **finite series** or the ***n*th partial sum** of the sequence and is denoted by: $a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^{n} a_i$.
  - The sum of all the terms of the **infinite sequence** is called an infinite series and is denoted by: $a_1 + a_2 + a_3 + \cdots + a_i + \cdots = \sum_{i=1}^{\infty} a_i$.

- **Properties of Sums**

  1. $\sum_{i=1}^{n} c = cn,$ c is a constant
  2. $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i,$ c is a constant
  3. $\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$
  4. $\sum_{i=1}^{n}(a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$

Section 2:

- **The nth term of an arithmetic sequence:** $a_n = dn + c$ where d is the common difference between consecutive terms of the sequence and $c = a_1 - d$.
  Recursion form for the nth term is $a_n = a_1 + (n-1)d$
- **The sum of a finite arithmetic sequence:** The sum of a finite arithmetic sequence with n terms is $S_n = \frac{n}{2}(a_1 + a_n)$.

Section 3:

- **The nth term of a geometric sequence**: $a_n = a_1 r^{n-1}$ where r is the common ratio of consecutive terms of the sequence.
- **The sum of a Finite Geometric Sequence**: The sum of the finite geometric sequence

$$a_1,\ a_1 r,\ a_1 r^2,\ a_1 r^3,\ a_1 r^4,\ \ldots,\ a_1 r^{n-1}$$

with common ratio $r \neq 1$ is given by $S_n = \sum_{i=1}^{n} a_1 r^{i-1} = a_1\left(\frac{1-r^n}{1-r}\right)$

- **The sum of an Infinite Geometric Series**: If $|r| < 1$, the infinite geometric series

$a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \ldots + a_1 r^{n-1} + \ldots$ has the sum $S = \sum_{i=0}^{\infty} a_1 r^i = \frac{a_1}{1-r}$.

Section 4:

- **Sums of Powers of Integers**:

1. $1 + 2 + 3 + 4 + \ldots + n = \frac{n(n+1)}{2}$
2. $1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$
3. $1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$
4. $1^4 + 2^4 + 3^4 + 4^4 + \ldots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
5. $1^5 + 2^5 + 3^5 + 4^5 + \ldots + n^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$