4.3 Right Triangle Trigonometry

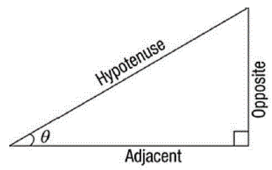
Right Triangle Definitions of Trig Functions

Let θ be an acute angle of a right triangle. The six trigonometric functions of the angle θ are defined as follows:

$$\sin\theta = \frac{opp}{hyp} \quad \cos\theta = \frac{adj}{hyp} \quad \tan\theta = \frac{opp}{adj}$$
$$\csc\theta = \frac{hyp}{opp} \quad \sec\theta = \frac{hyp}{adj} \quad \cot\theta = \frac{adj}{opp}$$

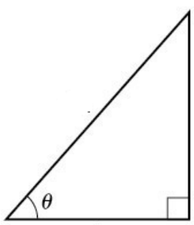
The abbreviations opp, adj, and hyp represent the lengths of the three sides of a right triangle.

opp = the length of the side *opposite* θ
adj = the length of the side *adjacent* to θ
hyp = the length of the *hypotenuse*



S O H C A H T O A

Ex a) Use the triangle to find the values of the 6 trig functions of θ.



Sine, Cosine and Tangent of Special Angles

*** These values can be found by looking at the unit circle **

$$\sin 30^\circ = \sin\frac{\pi}{6} = \frac{1}{2} \qquad \cos 30^\circ = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\sin 45^\circ = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \cos 45^\circ = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \tan 45^\circ = \tan\frac{\pi}{4} = 1$$

$$\sin 60^\circ = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \cos\frac{\pi}{3} = \frac{1}{2} \qquad \tan 60^\circ = \tan\frac{\pi}{3} = \sqrt{3}$$

<u>Cofunctions</u> (the 3 main trig functions and their inverses) of **complimentary** angles are equal. If θ is an acute angle, the following relationships are true. For example, $\sin 30° = \frac{1}{2} = \cos 60°$

$$\sin(90^\circ - \theta) = \cos\theta \qquad \cos(90^\circ - \theta) = \sin\theta$$
$$\tan(90^\circ - \theta) = \cot\theta \qquad \cot(90^\circ - \theta) = \tan\theta$$
$$\sec(90^\circ - \theta) = \csc\theta \qquad \csc(90^\circ - \theta) = \sec\theta$$

Fundamental Trig Identities

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$