1.9 Inverse Functions

Lets say $f(x) = x + 4$ and the values we found (if graphing) are {(1, 5), (2 6), (3, 7), (4, 8)}   by interchanging the x and y coordinates of these ordered pairs, we can form the inferse function of f, denoted f^{-1}. Then, the inverse of $f(x)$, $f^{-1}(x) = x - 4$ {(5, 1), (6, 2), (7, 3), (8, 4)}

Note that the domain of $f(x)$ is the range of $f^{-1}(x)$, and the range of $f(x)$ is the domain of $f^{-1}(x)$

How to find the inverse of a function:

1. Replace "$f(x)$" with y
2. Switch x and y
3. Solve for y
4. Replace "y" with the notation for inverse, $f^{-1}(x)$

Examples: Find the inverse of the following:

a.) $f(x) = 4x$

b.) $g(x) = -\frac{1}{3}x + 1$

> **Definition of Inverse Function**
>
> Let f and g be two functions such that
>
> $f(g(x)) = x$ for every x in the domain of g
>
> and
>
> $g(f(x)) = x$ for every x in the domain of f.
>
> Under these conditions, the function g is the **inverse function** of the function f. The function g is denoted by f^{-1} (read "f-inverse"). So,
>
> $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
>
> The domain of f must be equal to the range of f^{-1}, and the range of f must be equal to the domain of f^{-1}.

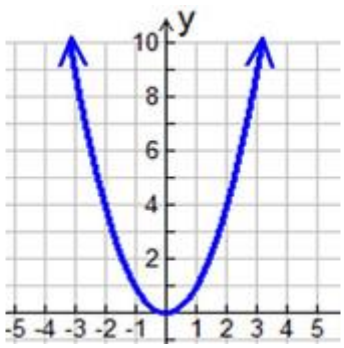
If the function g is the inverse function of the function f, it must also be true that the function f is the inverse function of the function g. For this reason, you can say that the functions and are inverse functions of each other.

c.) Which of the functions is the inverse function of $(x) = \frac{5}{x-2}$?

$$g(x) = \frac{x-2}{5} \qquad\qquad h(x) = \frac{5}{x} + 2$$

One-to-One Functions

By looking at a graph, you can determine whether or not it has an inverse function by preforming the horizontal line test. If, at any point, drawing a horizontal line would end up touching the graph in more than one place, it DOES NOT have an inverse function. In other words, if a function has repeating y values, it does not have an inverse function.



x	$f(x) = x^2$
-2	4
-1	$\boxed{1}$
0	0
1	$\boxed{1}$
2	4
3	9