

# 3.1 Exponential Functions and Their Graphs

## Part 2: The Natural Exponential & Applications

### A review of exponent properties:

Ex a) When comparing  $f(x) = 2^x$  and  $k(x) = 2^{-x}$ , how could you write  $k(x)$  in terms of  $f(x)$ ?

| PROPERTY                                                                               | EXAMPLE                                                                                                                               |
|----------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------|
| <b>Product of Powers</b><br>$a^m \cdot a^n = a^{m+n}$                                  | $x^2 \cdot x^7 \cdot x = x^{2+7+1} = x^{10}$<br>$3^2 \cdot 3^7 = 3^{2+7} = 3^9 = 19,683$                                              |
| <b>Power of a Power</b><br>$(a^m)^n = a^{mn}$                                          | $(n^3)^4 = n^{3 \cdot 4} = n^{12}$<br>$(4^2)^3 = 4^{2 \cdot 3} = 4^6 = 4,096$                                                         |
| <b>Power of Product</b><br>$(ab)^m = a^m b^m$                                          | $(xy)^5 = x^5 y^5$<br>$(42 \cdot 12)^2 = 42^2 \cdot 12^2 = 254,016$                                                                   |
| <b>Quotient of Powers</b><br>$\frac{a^m}{a^n} = a^{m-n}$                               | $\frac{x^{11}}{x^4} \cdot x^{11-4} = x^7$<br>$\frac{6^{12}}{6^8} = 6^{12-8} = 6^4 = 1,296$                                            |
| <b>Power of a Quotient</b><br>$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ | $\left(\frac{x}{y}\right)^5 = \frac{x^5}{y^5}$<br>$\left(\frac{x^2}{4y}\right)^3 = \frac{x^{2 \cdot 3}}{4^3 y^3} = \frac{x^6}{64y^3}$ |
| <b>Zero as an exponent</b><br>$a^0 = 1 \quad a \neq 0$                                 | $5^0 = 1$<br>$x^0 = 1$                                                                                                                |
| <b>Negative Exponents</b><br>$a^{-x} = \frac{1}{a^x}$                                  | $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$<br>$\frac{1}{y^{-7}} = y^7$                                                                    |

Rewrite each using properties of exponents.

Ex b)

Ex c)

Ex d)

Ex e)

Ex f)

Ex g) Using properties of exponents, which of these are equivalent?

### The One-to-One Property:

- For  $a > 0$  and  $a \neq 1$ ,  $a^x = a^y$  if and only if  $x = y$
- Used to solve exponential functions that:

- How to use the one-to-one property:

- Use the one-to-one property to solve each:

Ex h)

Ex i)

Ex j)

## Natural Base $e$

- $e$  is an irrational number (never ending and non-repeating decimal)
- $e \approx 2.718281828 \dots$
- This number,  $e$ , is called the natural base
- The function  $f(x) = e^x$  is called the natural exponential function
- There is a button for  $e$  on your calculator, and when pushed it prompts you to input an exponent ( $x$ )

Using your calculator, evaluate the following if  $f(x) = e^x$ . Round to the nearest thousandth.

Ex k)

Ex l)

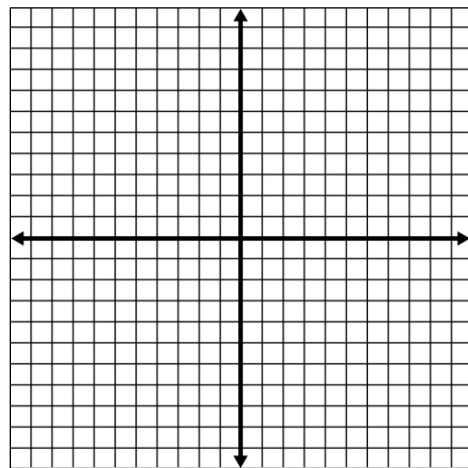
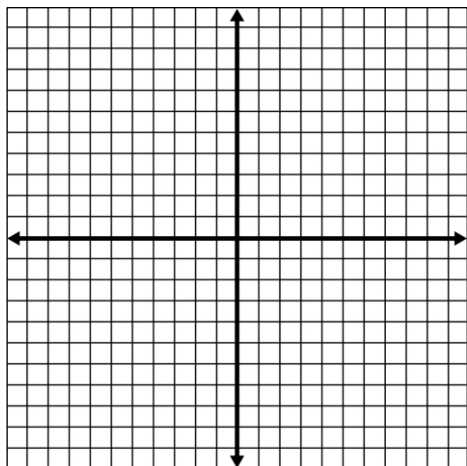
Ex m)

## ***Graphing Natural Exponential Functions***

Graph each natural exponential function (plot 4-6 points per function, to your best accuracy)

Ex n)

Ex o)



### ***Applications of Exponential Functions***

When do we use exponentials in real life?

By the time you're an adult... you'll use it for EVERYTHING!!!

- Buying or renting a car, buying or renting a house or apartment, student loans for college, lending others money if you have a large amount, investing in companies or stocks, etc!

## ***Compound Interest***

Compound interest:

Ex p)