3.2 Logarithmic Functions and their Graphs

Definition of a log function with base a: $f(x) = log_a(x)$ ← read as "log base a of x"

Logarithms are exponents $log_2 8 = 3$ because 2 must be raised to the 3rd power to get 8

$y = log_a x$ IS EQUIVALENT TO $x = a^y$

Logarithmic form Exponential form

Example 1) Change the following from logarithmic form to exponential form:

Logarithmic Form	Exponential Form
$log_2 8 = 3$ ⟷	$8 = 2^3$
$log_3 9 = 2$ ⟷	
$log_5 125 = 3$ ⟷	

Example 2) Evaluate:

a) $f(x) = log_2 x$ $if\ x = 32$ (ask yourself, "2 to what power equals 32? $2^? = 32$

b) $f(x) = log_4 x$ $if\ x = 2$

Common Logarithmic Function: log function with a base of 10

- Denoted log_{10} BUT is more commonly seen and written as log
- When you see log with no written base, it is always understood to have a base of 10
- If you plugged "log 2.5" into your calculator, it is evaluating $log_{10}(2.5)$

Properties of log functions:

1. $log_a 1 = 0$ ← because $a^0 = 1$ (anything to the power of zero equals 1)
   Ex) $log_3 1 = 0$ because $3^0 = 1$

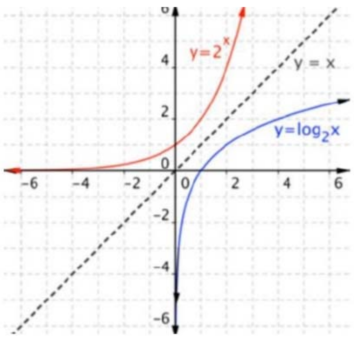
2. $log_a a = 1$ ← because $a^1 = a$ (anything to the first power equals itself)
   Ex) $log_7 7 = 1$ because $7^1 = 7$

3. $log_a a^x = x$ and $a^{log_a x} = x$ ← these two are inverses
   Ex) Simplify $6^{log_6 20} =$

4. If $log_a x = log_a y$, then $x = y$ ←both have the same bases ->we can use one-to-one property
   Ex) $log_3 x = log_3 12$ → x = 12
   Ex) $\log(2x + 1) = log x$ →

Graphs of Log functions:

$f(x) = a^x$ *and* $f(x) = log_a x$ ARE INVERSES→ they are reflections of each other about the line y=x



Graphs in the form $f(x) = a^x$

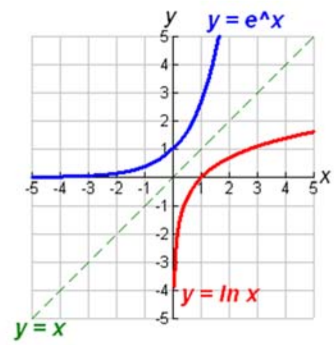
- Domain: (-∞,∞)
- Range: (0,∞)
- Y intercept: (0,1)
- Horizontal asymptote at y=0
- No vertical asymptote

Graphs in the form $log_a x$ *when* $a > 1$

- Domain: (0, ∞)
- Range: (-∞,∞)
- X intercept: (1,0)
- Increasing
- Has an inverse function
- Vertical asymptote at y=0
- No horizontal asymptote
- Continuous
- Reflection of the graph of $y = a^x$ about x=y

<u>The Natural Logarithmic Function</u>: $\ln x$ ← "the natural log of x" or "el n of x"

- $f(x) = log_e x$ = $\ln x$ when $x > 0$
- $\ln x$ is the INVERSE of e^x ← e^x and $\ln x$ reflect each other about the line y=x
- Written without a base, the base is ALWAYS e



Properties of the natural logarithm:

1. $\ln 1 = 0$ ← because $e^0 = 1$
   Ex) Evaluate $\frac{\ln 1}{3}$
2. $\ln e = 1$ ← because
   Ex) Evaluate $2\ln e$
3. $\ln e^x = x$ ← because
   Ex) Evaluate $e^{\ln 5}$
4. If $\ln x = \ln y$, then x = y ← they have the same base, so we can use the one-to-one property
   Ex) Evaluate $\ln(x - 1) = \ln 3$

Transformations of Graphs of Logarithmic Functions

Horizontal Translations

$\log_a(x-h)$ graph shifts to the right

$\log_a(x+h)$ graph shifts to the left

Vertical Translations

$\log_a(x)+k$ graph shifts up

$\log_a(x)-k$ graph shifts down

Axis flips

$\log_a(-x)$ graph flips over the y axis

$-\log_a(x)$ graph flips over the x axis