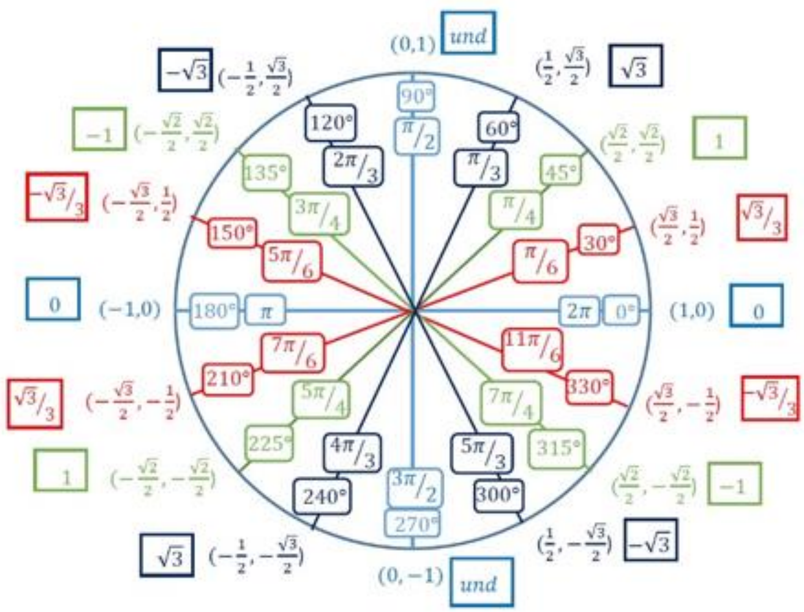
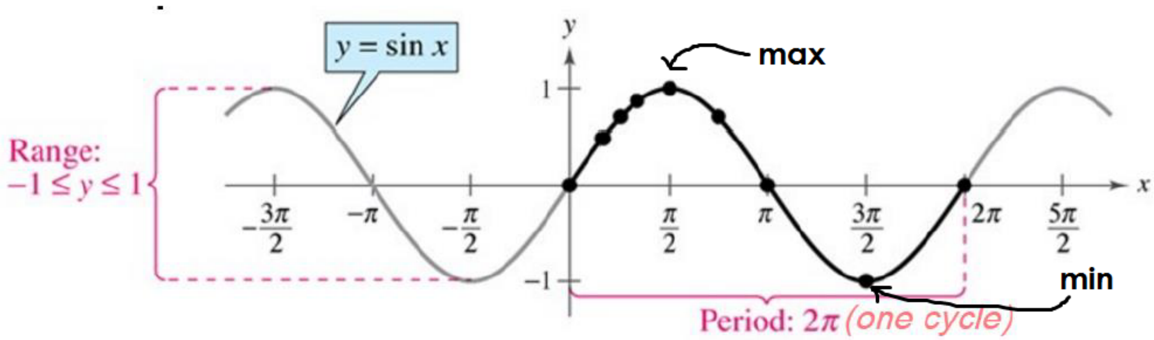
4.5 Graphs of Sine and Cosine Functions

- As you move along the graph of a trig function from left to right → the graph will repeat itself over and over
  - WHY? → Because the trig functions of an angle go through the same values every time we rotate the terminal side of the angle 360°, or 2π
  - Functions that repeat themselves like this are called <u>periodic functions</u> → all 6 trig functions are periodic functions
- Period of a trig function: the smallest interval over which it always repeats itself
  - All 6 trig functions repeat after each full revolution of 2π (360°), although tangent and cotangent repeat themselves over every half revolution of π (180°)



The graph of the sine function is a **sine curve** $f(x) = \sin x$ or $y = \sin x$.

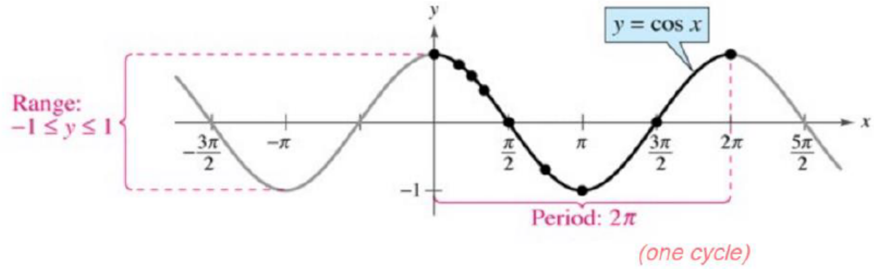


Some points on the graph of $y = \sin x$

	$\sin 0 = 0$ $\Downarrow$	$\sin \frac{\pi}{6} = \frac{1}{2} = 0.5$ $\Downarrow$	$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \approx 0.7$ $\Downarrow$	$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \approx 0.9$ $\Downarrow$	$\sin \frac{\pi}{2} = 1$ $\Downarrow$
(x, y)	$(0, 0)$	$\left(\frac{\pi}{6}, 0.5\right)$	$\left(\frac{\pi}{4}, 0.7\right)$	$\left(\frac{\pi}{3}, 0.9\right)$	$\left(\frac{\pi}{2}, 1\right)$

Note that the sine curve is symmetric with respect to the origin. Therefore, as we discussed before, the sine function is an odd function. For every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.

The graph of the cosine function is a **cosine curve** $f(x) = \cos x$ or $y = \cos x$.



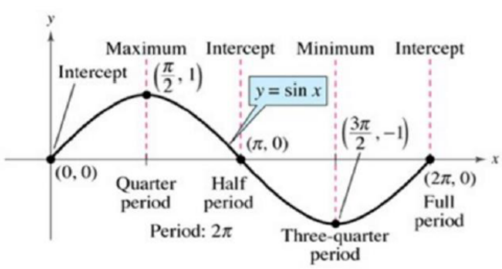
Some points on the graph of $y = \cos x$

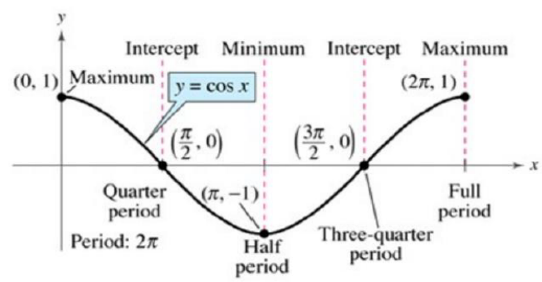
	$\cos 0 = 1$	$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \approx 0.9$	$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \approx 0.7$	$\cos\frac{\pi}{3} = \frac{1}{2} = 0.5$	$\cos\frac{\pi}{2} = 0$
	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
(x, y)	$(0, 1)$	$\left(\frac{\pi}{6}, 0.9\right)$	$\left(\frac{\pi}{4}, 0.7\right)$	$\left(\frac{\pi}{3}, 0.5\right)$	$\left(\frac{\pi}{2}, 0\right)$

Ex 1)

Note that the cosine curve is symmetric with respect to the y-axis. Therefore, as we discussed before, the sine function is an even function. For every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.

|To sketch the graphs of the basic sine and cosine functions by hand, it helps to note five key points in one period: the intercepts, maximum points, and minimum points.





We will look at the sine and cosine functions in the following forms. We will investigate what transformations occur given each of the constants.

$$y = d + a\sin(bx - c) \text{ and } y = d + a\cos(bx - c)$$

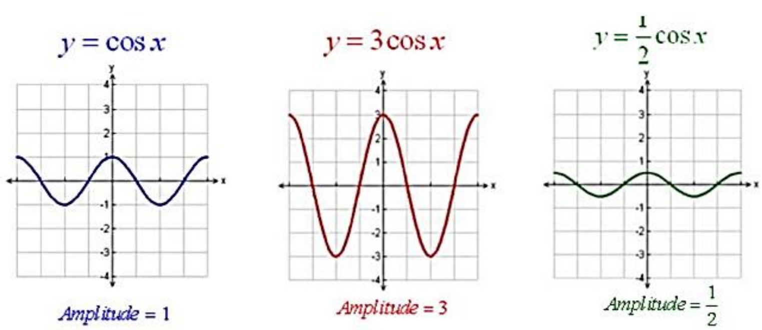
Definition of **Amplitude** of Sine and Cosine Curves

The amplitude of $y = a\sin x$ and $y = a\cos x$ represents half the distance between the maximum and minimum values of the function is given by Amplitude = $|a|$.

If $|a| > 1$, there is a vertical stretch. If $|a| < 1$, there is a vertical shrink.

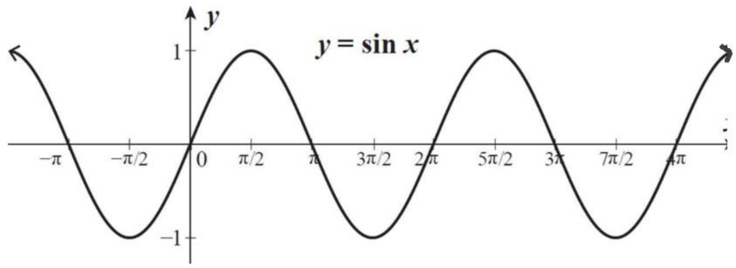
Amplitude is the vertical distance between the horizontal axis and the peak at the top of each wave

For example:

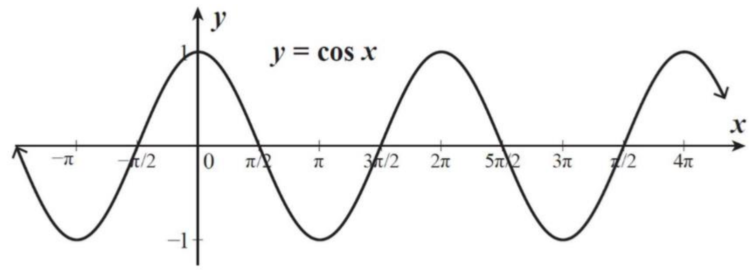


the amplitude of y = sin(x) and y = cos(x) is 1

Ex 1)



Ex 2)

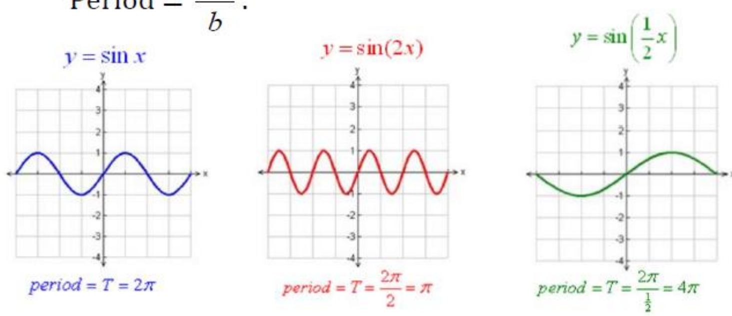


Period of Sine and Cosine Function

Let b be a positive real number. The period of $y = \sin bx$ and $y = \cos bx$ is given by

$$\text{Period} = \frac{2\pi}{b}.$$

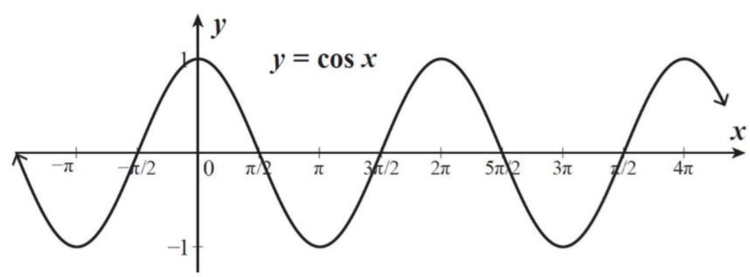
For example:



If $0 < b < 1$, the period is greater than 2π and represents a horizontal stretch.

If $b > 1$, the period is less than 2π and represents a horizontal shrink.

Ex 3)



Reflections of Sine and Cosine Functions

As before, $y = -f(x)$ is a reflection in the x-axis and $y = f(-x)$ is a reflection in the y-axis.

Translations of Sine and Cosine Curves

In this form $y = a\sin b(x - c) + d$ and $y = a\cos b(x - c) + d$, c represents a horizontal shift and d represents a vertical shift such that:

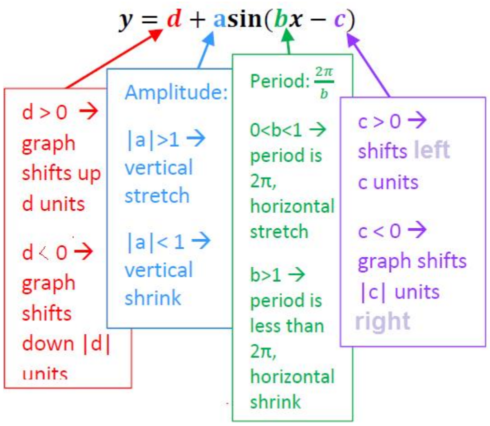
If $c > 0$, the graph shifts to the right c units. If $c < 0$, the graph shifts $|c|$ units to the left.

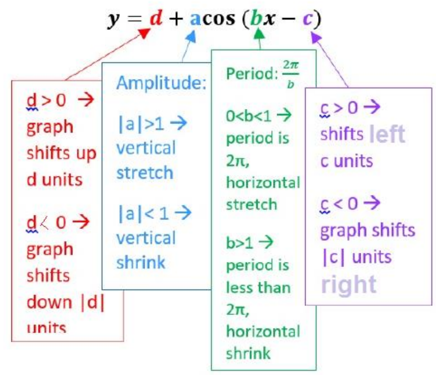
If $d > 0$, the graph shifts up d units. If $d < 0$, the graph shifts $|d|$ units down.

In this form $y = d + a\sin(bx - c)$ and $y = d + a\cos(bx - c)$, (assume $b > 0$), the left and right endpoints (horizontal shift) of a one-cycle interval can be determined by solving the equations $bx - c = 0$ and $bx - c = 2\pi$. d represents a vertical shift as stated above. If $d > 0$, the graph shifts up d units. If $d < 0$, the graph shifts $|d|$ units down.

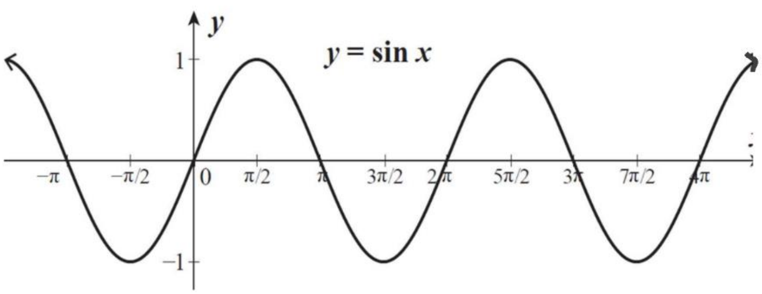
left

right

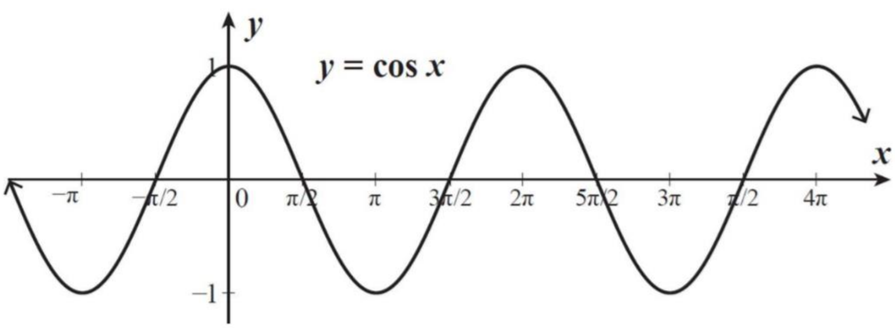




Ex 4)



Ex 5)



Ex 6)