Section 2 Logarithmic Functions

Definition of Logarithmic Function with Base a

For $x > 0$, $a > 0$, and $a \neq 1$,

$y = \log_a x$ if and only if $x = a^y$.

The function given by

$f(x) = \log_a x$ Read as "log base a of x

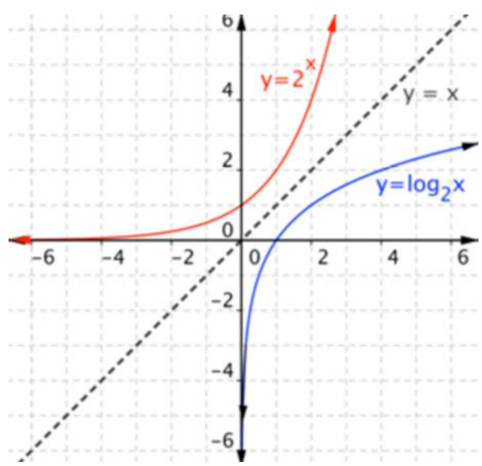
Is called the **logarithmic function with base** a.

The equation $y = \log_a x$ and $x = a^y$ are equivalent.

Properties of Logarithms

1. $\log_a 1 = 0$ because $a^0 = 1$.
2. $\log_a a = 1$ because $a^1 = a$.
3. $\log_a a^x = x$ and $a^{\log_a x} = x$. Inverse properties
4. If $\log_a x = \log_a y$, then $x = y$. One-to-One Property

To sketch the graph of $y = \log_a x$ you can use the fact that the graphs of inverse functions are reflections of each other in the line $y = x$.



- Domain: $(0, \infty)$
- Range: $(-\infty, \infty)$
- x-intercept: $(1,0)$
- Increasing
- One-to-one, therefore has an inverse function
- y-axis is a vertical asymptote $(\log_a x \to -\infty \quad as \quad x \to 0^+)$
- Continuous

Transformations of Graphs of Logarithmic Functions

Horizontal Translations

$\log_a(x-h)$ graph shifts to the right

$\log_a(x+h)$ graph shifts to the left

Vertical Translations

$\log_a(x)+k$ graph shifts up

$\log_a(x)-k$ graph shifts down

Axis flips

$\log_a(-x)$ graph flips over the y axis

$-\log_a(x)$ graph flips over the x axis

The Natural Logarithmic Function

The function defined by

$$f(x)=\log_e x=\ln x, \quad x>0$$

is called the natural logarithmic function.

The natural logarithmic function is the inverse of the exponential function.

$f(x)=e^x$ has an inverse function of $f(x)=\log_e x=\ln x$

Properties of Natural Logarithms

1. $\ln 1=0$ because $e^0=1.$
2. $\ln e=1$ because $e^1=e.$
3. $\ln e^x=x$ and $e^{\ln x}=x.$ Inverse Properties
4. If $\ln x=\ln y,$ then $x=y.$ One-to-One Property