Algebra 1 Concepts Review – Important items from the fundamentals unit

Adding and Subtracting Fractions/Rational Expressions

- YOU MUST FIND A COMMON DENOMINATOR! You cannot add/subtract these items unless their denominators are the SAME!!
- Once the denominators are the same, you only add/subtract the NUMERATORS – the denominator remains the same
- To find a common denominator of rational expressions being added or subtracted:
  1.) Factor every single denominator, if possible
  2.) The common denominator will be a combination of each different factor found between all of the denominators. It will be written as a product. Account for any repeating factors only once
  3.) Rewrite each fraction using the new denominator. Be sure to multiply each numerator by the same new factor(s) you multiplied their individual denominators by
  4.) Add/subtract the numerators, keep the denominator the same

To solve an equation with rational expressions on both sides of the equals sign:

1.) Factor the denominators of each rational expression found within the equation
2.) Combine all of the different factors found between all denominators and write it as a product
3.) Multiply EVERY SINGLE piece by this product found in step 2, called your common multiplier. Do not "actually multiply" anything by distributing or foiling, but instead leave factors written as a product and cancel common factors
4.) You now have an equation without fractions. Solve accordingly

Multiplying Fractions/Rational Expressions:

- You do not need to make ANY changes to either fraction!
  1.) Factor the numerator and denominator of each to make this process a lot easier
  2.) Multiply straight across → numerator × numerator and denominator × denominator. Do not "actually multiply," but instead write the numerator and denominator of your answer written as the product of factors
  3.) Cancel factors found in both the numerator and denominator – you can only cancel ONE PAIR at a time!

Dividing Fractions/Rational Expressions:

- You do not need to make ANY changes to either fraction!
- You can either cross multiply, or "keep, change, flip"
- To keep change flip you:
  1.) Rewrite (keep) the first fraction as is
  2.) change the division sign to multiplication
  3.) flip the second fraction (switch the position of the numerator and denominator)
  4.) now follow the steps above and multiply the fractions!

Absolute Value Equations:

- a number in front of absolute value bars, like the 2 in $2|3x+1|$ is a coefficient, but CANNOT be distributed through to the inside of the absolute value bars!! No movement or value changes can occur while those bars are still there
  1.) isolate the absolute value part of the equation by using opposite operations to move all other items to the other side
  2.) Now, you can drop the absolute value bars and set up 2 equations. The signs on the terms that were once inside of the bars WILL STAY EXACTLY THE SAME for both equations. However, one equation will have the positive version of the number on the other side of the equals sign, and one will have the negative versio

Absolute Value Inequalities:

1.) isolate the absolute value part of the inequality by using opposite operations to move all other items to the other side
2.) Now, you can drop the absolute value bars and set up 2 inequalities. The signs on the terms that were once inside of the bars WILL STAY EXACTLY THE SAME for both equations. However, one equation will have the positive version of the number on the other side of the equals sign, and one will have the negative version along with a FLIPPED inequality sign

Equations with Radicals:

- No matter how many radical terms an equation has, the process is going to be the same. Remember opposite operations with radicals – squaring a square root gets rid of the root, as does cubing a cubed root, and so on
  1.) If there is only one radical term, isolate it onto one side of the equals sign and move everything else over using opposite operations. If there is more than one radical term, choose one to isolate, and move others over with all of the other terms
  2.) Raise each side of the equation to the same power as the index of the radical. Make sure you realize that when raising any POLYNOMIAL (2 terms or more) to a power, you DO NOT distribute the exponent. You must actually multiply that polynomial by itself as many times as the exponent calls for
  3.) If your equation originally had more than one radical, after step 2 you will only have successfully gotten rid of one. So, repeat steps 1-2 for remaining radicals as many times as necessary
- You must always check if your solution(s) are **EXTRANEOUS!** Meaning that they were solutions you should have gotten, however when plugged in, they do not work within the equation

Equations with Rational Exponents (exponents that are fractions – think how fraction and ration rhyme):

- Rational exponents are used as another way to represent radical terms. The numerator of the fraction represents the exponent on the base, and the denominator represents the index of the radical the base will be inside of
- Remember that it is not proper to express anything using rational exponents. If you see them occur within an equation, rewrite the terms as radicals. If they occur along with many other variable terms with exponents, you may want to apply exponent properties first to simplify your expression down
- Remember opposite operations with radicals – squaring a square root gets rid of the root, as does cubing a cubed root, and so on

Types of Solutions when Solving Equations:

- Extraneous solutions:
    - Solutions that you should have gotten from solving, but when you plug them back into the original equation, they do not work either by resulting in a "false" statement, or be resulting in something that does not exist
    - Always check your solutions of equations with radicals and equations with rational terms, they may be extraneous
    - These solutions don't work because they are excluded from the domain of the original function

- "No solution:" **ø**
    - When solving an equation, and either the variable terms "go away" or the numerical terms "go away," and you are left with a false statement
- "All Real Numbers" $\mathbb{R}$
    - When solving an equation, at some point you end up with a more condensed equation where the items on each side of the equals sign are exactly the same
    - Plug in a few different values for the variable and test that they all will give you a true statement!

Making sure your answer for anything is as simplified as possible:

- Items you CANNOT have in a solution:
    - Rational exponents – they really don't make much sense as is, so be sure to rewrite terms using radicals
    - Negative exponents – these definitely don't make sense! To make negative exponents positive, move it and its base (ONLY) to the opposite location it is currently in within the fraction, and change the exponent to positive
    - Radicals in the denominator of a fraction – you must rationalize! (see below)
- Simplifying rational answers (answers involving fractions)
    - Fractions with a monomial in the numerator and denominator can be simplified by dividing out a GCF of the numerical terms, and by using exponent properties to combine any of the same variables
    - If the numerator and/or denominator involves a polynomial – BE CAREFUL! You can only divide out a common term/factor if it can be divided out of EVERY SINGLE TERM within the ENTIRE FRACTION!!! Otherwise, you cannot simplify. This is a very easy mistake to make. NEVER DIVIDE A NUMBER OUT OF ANY TERMS INSIDE OF RADICALS!! Only the numbers/terms on the outside

Radicals:

- Simplifying:
    - If a radical is not a perfect root of the index (like),
        1.) try to find 2 numbers that you can multiply together where one IS a perfect root of the index
        2.) rewrite the number inside of the radical as a product using the 2 numbers found above
        3.) take the root of the number that is a perfect root, and write its root on the outside of the radical, leaving the other number on the inside
    - When there are variable terms with exponents on the inside- remember that when the index is the same as the exponent on an item, both pieces essentially "cancel" out and you are left with the terms that were inside of the radical
        1.) Note what the index is, and try to rewrite each variable term so that one or more have the same exponent as the index
        2.) Take the root of those terms and write it outside of the radical
- Adding and subtracting:
    - You can only add or subtract radical terms if the radical portion of the terms are exactly the same, index included.
        1.) SIMPLIFY EACH FIRST!! This will help determine which terms you can combine (add or subtract)
        2.) Add or subtract the coefficients only of any terms that have the exact same radical portion. Leave the readical part the same (similar to properties of adding/subtracting variable terms)
- Multiplying:
    - You can multiply any radicals that have the same index
        1.) SIMPLIFY EACH FIRST!!! This will save you time and will give you smaller numbers to work with
        2.) Multiply their coefficients if they have them, as well as the terms on the inside. Remember, if you are multiplying variable terms that are the same, we ADD their exponents
        3.) Simplify your answer
- Dividing (Fractions with Radicals in the Denominator – sometimes they are in the numerator too!)
  WE CANNOT HAVE A RADICAL IN THE DENOMINATOR OF A FRACTION EVER! We need to rationalize these fractions to make them into valid fractions. There are 2 different instances in which we could see a radical in the denominator:
    - When there is a MONOMIAL in the denominator that involves a radical:
        1.) Multiply the top and bottom of the fraction by the radical in the denominator and simplify

    - When there is a BINOMIAL in the denominator of a fraction:
        1.) Multiply the numerator and denominator by the CONJUGATE of the denominator, which is a binomial with the exact same terms, except for the opposite sign in between them. To multiply, you will have to distribute and/or FOIL, so take your time. simplify