Chapter 4 Section 2
Trigonometric Functions: The Unit Circle

Definitions of Trigonometric Functions

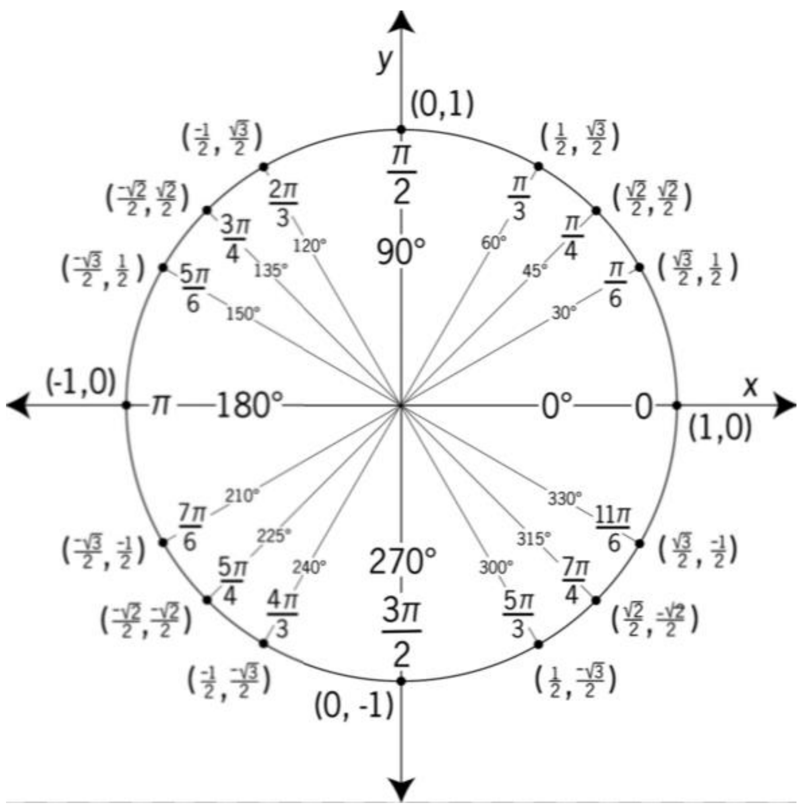
Let t be a real number and let (x, y) be a point on the unit circle corresponding to t.

$\sin t = y$ $\qquad$ $\cos t = x$ $\qquad$ $\tan t = \dfrac{y}{x}, x \neq 0$

$\csc t = \dfrac{1}{y}, y \neq 0$ $\qquad$ $\sec t = \dfrac{1}{x}, x \neq 0$ $\qquad$ $\cot t = \dfrac{x}{y}, y \neq 0$

The Unit Circle

The unit circle is given by $x^2 + y^2 = 1$. The real number line is wrapped around the unit circle so that each real number t corresponds to a point (x, y) on the circle. Also, each real number t corresponds to a central angle θ (in standard position) whose radian measure is t. The real number t is the length of the arc intercepted by the angle θ, given in radians.



Definition of Periodic Function

A function f is periodic if there exists a positive real number c such that

$$f(t+c) = f(t)$$

for all t in the domain of f. The smallest number c for which f is periodic is called the period of f.

Since $\sin(t+2\pi n) = \sin(t)$ and $\cos(t+2\pi n) = s\cos(t)$ for any integer n and real number t. Sine and cosine are periodic functions.

Even and Odd Trigonometric Functions

The cosine and secant functions are *even*.

$$\cos(-t) = \cos t \qquad \sec(-t) = \sec t$$

The sine, cosecant, tangent, and cotangent functions are *odd*.

$$\sin(-t) = -\sin t \qquad \csc(-t) = -\csc t$$
$$\tan(-t) = -\tan t \qquad \cot(-t) = -\cot t$$