

## Asymptotes of exponential and logarithmic functions and how to find them:

- Exponential functions: ONLY have horizontal asymptotes
  - How to find it → if a number is being added or subtracted from the base in which  $x$  is part of the exponent, that number (and its sign) is where there is a horizontal asymptote
  - Ex)  $3^{x-7} + 4$  horizontal asymptote is at  $y = 4$
  - Ex)  $5^{x+1} - 1$  horizontal asymptote is at  $y = -1$
- Logarithmic functions: ONLY have vertical asymptotes
  - How to find it → set what is inside of the parentheses equal to zero, and solve for  $x$ , that number (and sign) is your vertical asymptote
  - Ex)  $\log_2(x - 1)$   $x - 1 = 0 \rightarrow x = 1$  vertical asymptote is at  $x = 1$
  - Ex)  $\log(x + 3)$   $x + 3 = 0 \rightarrow x = -3$  vertical asymptote is at  $x = -3$

The domain of ANY exponential function:  $(-\infty, \infty)$  ALWAYS!!!!!!

- Ex) Any exponential function like:  $f(x) = 7^x$ ,  $g(x) = e^{2x}$ ,  $k(x) = 3^{8x-4}$  have a domain of  $(-\infty, \infty)$

Inverses of exponentials and their graphs:  $f(x) = a^x$  and  $g(x) = \log_a x$  are inverses.

How to find the inverse of an exponential function:

1. Set the function equal to  $y$
2. Isolate the part of the function that includes the  $x$  on one side of the equals sign
3. Swap  $x$  and  $y$
4. Rewrite the exponential and the rest of the function as a logarithm (take the log of both sides- this will allow you to get the  $y$  out of the exponent)
5. Solve for  $y$

Ex) Find the inverse of  $k(x) = 7^{x+1} - 3$

1.  $y = 7^{x+1} - 3$
2.  $y + 3 = 7^{x+1}$
3.  $x + 3 = 7^{y+1}$
4.  $\log_7(x + 3) = 7^{\log_7 y + 1}$
5.  $\log_7(x + 3) = y + 1$
6.  $\log_7(x + 3) - 1 = y$  we call the inverse of  $k(x)$ :  $k'(x)$

## How to graph $k(x)$ and its inverse

For  $k(x) = 7^{x+1} - 3$

1. Figure out what the asymptote is, and draw it on the graph
  - $k(x)$  is an exponential, so it has a horizontal asymptote at  $y = -3$
2. Choose a few (easy) points to plug in for  $x$  and solve for  $y$  to get a few points to plot

$x$	$y$
0	4
-1	-2

For the inverse of  $k(x)$ , which is denoted  $k'(x)$   $k'(x) = \log_7(x + 3) - 1$

1. Figure out what the asymptote is, and draw it on the graph
  - $k'(x)$  is a logarithm, so its vertical asymptote is found by solving  $x + 3 = 0$ , and we get  $x = -3$  (or, you could note that since  $k(x)$  has a *horizontal asymptote* at  $y = -3$ ,  $k'(x)$  must have a *vertical asymptote* at  $x = -3$  since the two are inverses)
2. Instead of plugging  $x$  values in, we can just take the coordinates we found for  $k(x)$  and switch the  $x$ 's and  $y$ 's in each coordinate. (we can do this because  $k'(x)$  is the inverse of  $k(x)$ )

$x$	$y$
4	0
-2	-1

