Chapter 2: Polynomial Functions

2-1: Quadratic Functions

Sketch a quadratic function giving the following information: axis of symmetry, vertex, *x*-intercepts (if they exist) and *y*-intercept.

Examples:

1) $f(x) = -x^2 - 4x + 5$

2) $f(x) = \frac{1}{2}x^2 + 2x - 1$

Write a quadratic function given the vertex and a point on the graph.

Examples:

3) Vertex (3, 4): point (1, 2)

4) Vertex $(-\frac{1}{4}, \frac{3}{2})$: point (-2, 0)

Rewrite a quadratic in standard form $f(x) = ax^2 + bx + c$ into vertex form $f(x) = a(x-h)^2 + k$.

Examples:

5) $f(x) = x^2 - 8x - 20$

6) $f(x) = 3x^2 + 18x + 29$

7) $f(x) = -2x^2 + 8x - 11$

2-2: Polynomial Functions of Higher Degree

Sketch a polynomial of nth degree using the leading coefficient, the number of zeros, and multiplicity of the zeros.

Examples:

Sketch a function based on the following characteristics.

1) Degree 5 with positive leading coefficient and 3 real zeros (not repeating)
2) Degree 6 with negative leading coefficient and 3 real zeros (all 3 repeating)
3) Degree 4 with positive leading coefficient and 2 real zeros (neither repeating)

Sketch the graph of a polynomial by finding the zeros and using the sign of the leading coefficient.

Examples:

Graph the polynomial without the aid of a calculator.

4) $f(x) = x^3 - x^2 - x + 1$
5) $g(x) = -x^4 + 13x^2 - 36$

2-3: Polynomial Long and Synthetic Division

Use long or synthetic division to divide polynomials. Be sure to use placeholders when using long or synthetic division.

Use long division when the divisor is greater than degree 1.

Remainder theorem says that when dividing f(x) by (x – k), the remainder is f(k). (k, f(k)) is on the graph of f(x).

If there is no remainder, then (x – k) is a factor of f(x) and k is a zero.

Given one or more factors or zeros, find the remaining zeros by using long or synthetic division to reduce the polynomial to get a quadratic. Then solve by factoring, completing the square or using the quadratic formula.

Examples:

6) $h(x) = x^4 - 10x^3 + 6$
   a) Divide by (x – 10) and state the quotient.
   b) Is the binomial a factor of the polynomial?
   c) Find $h(10)$

7) $j(x) = 4x^4 - 10x^3 - 36x^2 - 50x - 48$
   a) Divide by $(4x + 6)$ and state the quotient.
   b) Is the binomial a factor of the polynomial?
   c) Find $h(-\frac{3}{2})$

Given a polynomial and one of its factors, find the remaining factors and all of the zeros.

8) $f(x) = x^3 + 8x^2 + 25x + 26\ ;\ x + 2$

9) $g(x) = 3x^3 + 10x^2 - 22x - 5\ ; 3x - 5$

Given a function and one of its zeros, find all of the zeros of the function.

10) $f(x) = x^5 - 5x^4 - 9x^3 + 45x^2 + 14x - 70\ ;\ \sqrt{7}$

11) $g(x) = x^5 + 2x^4 + 9x^3 + 18x^2 + 20x + 40\ ;\ -2$

12) $h(x) = x^4 + 3x^3 - 5x^2 - 21x + 22\ ;\ -3 + i\sqrt{2}$

13) $j(x) = x^4 - 2x^3 + 14x^2 - 8x + 40\ ; 1 + 3i$

2-4: Complex Numbers

Complex numbers come in pairs. If 3i is a zero, then -3i is also a zero. If $1 - i\sqrt{2}$ is a zero, then $1 + i\sqrt{2}$ is also a zero.

2-5: Zeros of Polynomial Functions

Every nth degree polynomial function has precisely n zeros in the complex number system.
Find a polynomial function given the zeros. Write as a product of linear factors and multiply.
Rational Zero Test (Rational Root Theorem): Possible rational zeros are p/q, where p are the factors of the constant and q are the factors of the leading coefficient.
You can list the possible rational zeros, test each one by plugging it in or using synthetic division, **OR** you can use your calculator to find any EXACT zeros. Use those zeros to reduce the polynomial to a quadratic to solve completely.

Find all of the zeros of each function.

14) $f(x) = 6x^3 - 3x^2 - 39x - 18$
15) $g(x) = x^3 - 4x^2 - 2x$
16) $h(x) = x^3 - 6x^2 + 13x - 10$
17) $g(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$
18) $j(x) = x^6 + 5x^4 - 9x^2 - 45$

Write the polynomial of least degree with integral coefficients that has the given zeros.

19) $0, 4, i$
20) $2 + 3i, 2, 3$

Practice problems in your book:
pages 208-210, problems 3 – 12, 15 – 18, 47 – 52, 61 – 64, 91– 102, 103 – 106
page 212 #4 – 7, 10 – 13
Check the odd problems in the back of your book. You do not need to do all of these problems, just get a feel for what you need to practice.