

Fundamentals of Algebra ^{test 2018} Review ^{Key}

1.) $-9n^2 + 3n + 2$

$$\begin{aligned} & -18 \quad 6 \cdot -3 = -18 \\ & 6 + -3 = 3 \\ & -9n^2 - 3n + 6n + 2 \\ & -(9n^2 + 3n - 6n - 2) \\ & -[(9n^2 + 3n) - (6n - 2)] \\ & -[3n(3n+1) - 2(3n+1)] \\ & -(3n-2)(3n+1) \end{aligned}$$

2.) $\frac{z^2 - z - 6}{z^2 + 6z + 9} \cdot \frac{z+3}{z^2 - 4}$

$$\frac{(z-3)(z+2)}{(z+3)(z+3)} \cdot \frac{(z+3)}{(z-2)(z+2)}$$

$$\frac{(z-3)(z+2)(z+3)}{(z+3)(z+3)(z-2)(z+2)}$$

$$\frac{z-3}{(z+3)(z-2)} \text{ or } \frac{z-3}{z^2+z-6}$$

3.) $6 - \frac{5}{t+3}$

$$\frac{6(t+3) - 5}{t+3} = \frac{6t+18-5}{t+3}$$

$$\frac{6t+13}{t+3}$$

4.) $-\frac{1}{y} + \frac{2}{y^2+1} + \frac{1}{y^3+y}$

$$-\frac{1}{y} + \frac{2}{y^2+1} + \frac{1}{y(y^2+1)}$$

LCD: $y(y^2+1)$

$$\frac{-1(y^2+1) + 2y + 1}{y(y^2+1)}$$

$$\frac{-y^2 - 1 + 2y + 1}{y(y^2+1)} = \frac{-y^2 + 2y}{y(y^2+1)} = \frac{y(2-y)}{y(y^2+1)} = \frac{2-y}{y^2+1}$$

5.) $\frac{\frac{x-4}{\frac{x}{4} - \frac{4}{x}}}{\frac{x^2-16}{4x}} = \frac{x-4}{\frac{x^2-16}{4x}} = \frac{x-4}{1} \cdot \frac{4x}{x^2-16} = \frac{4x(x-4)}{(x-4)(x+4)} = \frac{4x}{x+4}$

6.) $\frac{\sqrt{3}+1}{2\sqrt{2}-4} \cdot \frac{(2\sqrt{2}+4)}{(2\sqrt{2}+4)} = \frac{(\sqrt{3}+1)(2\sqrt{2}+4)}{(2\sqrt{2}-4)(2\sqrt{2}+4)}$

$$\frac{2\sqrt{6} + 4\sqrt{3} + 2\sqrt{2} + 4}{4\sqrt{2} + 8\sqrt{2} - 8\sqrt{2} - 16} = \frac{2\sqrt{6} + 4\sqrt{3} + 2\sqrt{2} + 4}{-8}$$

$$\frac{2\sqrt{6} + 4\sqrt{3} + 2\sqrt{2} + 4}{-8} = \frac{\sqrt{6} + 2\sqrt{3} + \sqrt{2} + 2}{-4}$$

7.) $\frac{(2x^3y^4)^3(16x^{-2}y^6)^{-3}}{(-3x^{-5}y^2)^2}$

$$\frac{2^3x^9y^{12}16^{-3/2}x^{-3}y^{-9}}{-3^2x^{-10}y^4} = \frac{8x^9y^{12}16^{-3/2}x^{-3}y^{-9}}{9x^{-10}y^4}$$

$$\frac{8x^{12}y^316^{3/2}}{9x^{-10}y^4} = \frac{8x^{12}y^3x^{15}}{9y^416^{3/2}}$$

$$\frac{8x^{27}y^3}{9y^4(64)} = \frac{8x^{27}y^3}{576y^4} = \frac{8x^{27}y^{-1}}{576}$$

$$\frac{x^{27}}{72y}$$

8.) $\sqrt[3]{16a^7bc^5} \cdot \sqrt[3]{32a^7b^5c^3}$

$$\sqrt[3]{8 \cdot 2 \cdot a^6 \cdot a \cdot b \cdot c^5} \cdot \sqrt[3]{8 \cdot 4 \cdot a^6 \cdot a^3 \cdot b^5 \cdot b^2 \cdot c^3}$$

$$2a^2c \sqrt[3]{2abc^5} \cdot 2a^2bc \sqrt[3]{4ab^7c^3}$$

$$4a^4c^2 \sqrt[3]{8a^2b^8c^8} = 8a^4bc^2 \sqrt[3]{a^2c^2}$$

9.) $2\sqrt{50} - \sqrt[3]{54} + \sqrt{32}$

$$2\sqrt{5 \cdot 2} - \sqrt[3]{27 \cdot 2} + \sqrt{16 \cdot 2}$$

$$2(5)\sqrt{2} - 3\sqrt[3]{2} + 4\sqrt{2}$$

$$10\sqrt{2} - 3\sqrt[3]{2} + 4\sqrt{2}$$

$$14\sqrt{2} - 3\sqrt[3]{2}$$

$$10.) -9m^2 + 3m + 2$$

$$\begin{array}{r} -18 \quad 6 \quad -3 \\ -9m^2 - 3m + 6m + 2 \\ -3m(3m+1) + 2(3m+1) \\ (-3m+2)(3m+1) = 0 \\ -3m+2=0 \quad 3m+1=0 \\ m = \frac{2}{3} \quad m = -\frac{1}{3} \end{array}$$

$$11.) 25x^2 - 10x + 1 = 0$$

$$\begin{array}{r} -5 \quad 25 \\ -5 \cdot -5 = 25, -5 + -5 = -10 \\ (25x^2 - 5x) - (5x + 1) = 0 \\ 5x(5x-1) - (5x+1) = 0 \\ (5x-1)(5x-1) = 0 \\ 5x-1=0 \\ x = \frac{1}{5} \end{array}$$

$$12.) x - 3(2x+3) = 8 - 5x$$

$$\begin{array}{r} x - 6x - 9 = 8 - 5x \\ -5x - 9 = 8 - 5x \\ +5x \quad +5x \\ -9 = 8 \\ \text{No solution} \end{array}$$

$$13.) \frac{2}{(a-4)(a-2)} = \frac{1}{a-4} + \frac{2}{a-2}$$

$$\text{LCD: } (a-4)(a-2)$$

$$\frac{(a-4)(a-2)}{(a-4)(a-2)} \cdot \frac{2}{(a-4)(a-2)} = \frac{(a-4)(a-2)}{(a-4)(a-2)} \cdot \frac{1}{(a-4)} + \frac{(a-4)(a-2)}{(a-4)(a-2)} \cdot \frac{2}{(a-2)}$$

$$2 = 1(a-2) + 2(a-4)$$

$$2 = a - 2 + 2a - 8 \rightarrow 2 = 3a - 10 \rightarrow 12 = 3a$$

$$a = 4, 4 \text{ is an extraneous solution,}$$

No solution

$$14.) (x+2)^2 - x^2 = 4(x+1)$$

$$(x+2)(x+2) - x^2 = 4x+4$$

$$x^2 + 4x + 4 - x^2 = 4x+4$$

$$4x+4 = 4x+4$$

$\mathbb{R}$

$$15.) 2b^2 - b - 1 = 0$$

$$-2 \cdot -1 = 2$$

$$2b^2 - 2b + b - 1 = 0 \rightarrow (2b^2 - 2b) + (b - 1) = 0$$

$$2b(b-1) + (b-1) = 0 \rightarrow (2b+1)(b-1) = 0$$

$$b = -\frac{1}{2}$$

$$b = 1$$

$$16.) (x-5)^{3/2} = 8$$

$$\sqrt{(x-5)^3} = 8$$

$$(\sqrt{(x-5)^3})^2 = (8)^2$$

$$(x-5)^3 = 64$$

$$(x-5)(x-5)(x-5) = 64$$

$$(x-5)(x^2 - 10x + 25) = 64$$

$$x^3 - 10x^2 + 25x - 5x^2 + 50x - 125 = 64$$

$$x^3 - 15x^2 + 75x - 189 = 64$$

$$(x^3 - 15x^2) + (75x - 189) = 0$$

NOT

on

test

(have not talked about xth root +hm)

$$17.) 3h(h-1)^{1/2} + 2(h-1)^{3/2} = 0$$

$$3h\sqrt{h-1} + 2\sqrt{(h-1)^3} = 0$$

$$3h\sqrt{h-1} + 2\sqrt{(h-1)^2(h-1)} = 0$$

$$3h\sqrt{h-1} + 2(h-1)\sqrt{h-1} = 0$$

$$\sqrt{h-1} [3h + 2(h-1)] = 0$$

$$\sqrt{h-1} [3h + 2h - 2] = 0$$

$$\sqrt{h-1} (5h - 2) = 0$$

$$\sqrt{h-1} = 0$$

$$5h - 2 = 0$$

$$(\sqrt{h-1})^2 = 0^2$$

$$5h = 2$$

$$h - 1 = 0$$

$$h = \frac{2}{5}$$

$$h \neq 1$$

$$18.) \begin{aligned} 3/|x-2| - 10 &= 11 \\ 3/|x-2| &= 21 \\ |x-2| &= 7 \end{aligned}$$

$$\begin{aligned} \swarrow \quad \searrow \\ x-2 &= 7 \quad x-2 = -7 \\ x &= 9 \quad x = -5 \end{aligned}$$

check

$$\begin{aligned} |9-2| &= 7 & |-5-2| &= 7 \\ |7| &= 7 \checkmark & |-7| &= 7 \checkmark \end{aligned}$$

$$\begin{aligned} 19.) (x-6)^2 - (x-2)^2 &= -3(x-4) \\ [(x-6)(x-6)] - [(x-2)(x-2)] &= -3x+12 \\ x^2-12x+36 - (x^2-4x+4) &= -3x+12 \\ x^2-12x+36-x^2+4x-4 &= -3x+12 \\ -8x+32 &= -3x+12 \\ -8x+20 &= -3x \\ 20 &= 5x \\ x &= 4 \end{aligned}$$

$$\begin{aligned} 20.) \sqrt{3x-2} - \sqrt{10-x} &= 2 \\ \sqrt{3x-2} &= 2 + \sqrt{10-x} \\ (\sqrt{3x-2})^2 &= (2 + \sqrt{10-x})^2 \\ 3x-2 &= (2 + \sqrt{10-x})(2 + \sqrt{10-x}) \\ 3x-2 &= 4 + 2\sqrt{10-x} + 2\sqrt{10-x} + 10-x \\ 3x-2 &= 14 - x + 4\sqrt{10-x} \\ 4x-16 &= 4\sqrt{10-x} \\ (4x-16)^2 &= (4\sqrt{10-x})^2 \\ (4x-16)(4x-16) &= 16(10-x) \\ 16x^2-128x+256 &= 160-16x \\ 16x^2-112x+96 &= 0 \\ a=16 \quad b=-112 \quad c=96 \end{aligned}$$

$$x = \frac{-(-112) \pm \sqrt{(-112)^2 - 4(16)(96)}}{2(16)}$$

$$x = \frac{112 \pm \sqrt{12544 - 6144}}{32}$$

$$x = \frac{112 \pm \sqrt{6400}}{32} = \frac{112 \pm 80}{32}$$

$$\begin{aligned} \frac{112+80}{32} &= \frac{192}{32} = 6 \\ \frac{112-80}{32} &= \frac{32}{32} = 1 \end{aligned}$$

$$\begin{aligned} 21.) 4x^2 + 16x &= 17 \\ 4x^2 + 16x - 17 &= 0 \\ a=4 \quad b=16 \quad c=-17 \\ x &= \frac{-16 \pm \sqrt{(16)^2 - 4(4)(-17)}}{2(4)} \end{aligned}$$

$$\begin{aligned} x &= \frac{-16 \pm \sqrt{256 + 272}}{8} \\ x &= \frac{-16 \pm \sqrt{528}}{8} = \frac{-16 \pm \sqrt{16 \cdot 33}}{8} \\ x &= \frac{-16 \pm 4\sqrt{33}}{8} = \frac{-4 \pm \sqrt{33}}{2} \end{aligned}$$

$$\begin{aligned} 22.) \frac{x}{x^2-4} + \frac{2}{3} &= \frac{1}{3x+6} \\ \frac{x}{x^2-4} + \frac{2}{3} &= \frac{1}{3(x+2)} \\ \text{LCM: } 3(x-2)(x+2) \end{aligned}$$

$$\begin{aligned} 3(x-2)(x+2) \left[\frac{x}{(x-2)(x+2)} + \frac{2}{3} \right] &= 3(x-2)(x+2) \left[\frac{1}{3(x+2)} \right] \\ x + 2(x-2)(x+2) &= 3(x-2)(x+2) \left[\frac{1}{3} \right] \end{aligned}$$

$$\begin{aligned} 3x + 2(x-2)(x+2) &= 3(x-2)(x+2) + x-2 \\ 3x + 2(x^2-4) &= 3(x^2-4) + x-2 \\ 3x + 2x^2 - 8 &= 3x^2 - 12 + x - 2 \\ 2x^2 + 3x - 8 &= 3x^2 + x - 14 \\ x^2 - 2x - 6 &= 0 \quad a=1 \quad b=-2 \quad c=-6 \end{aligned}$$

$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-6)}}{2(1)} = \frac{2 \pm \sqrt{4+24}}{2} = \frac{2 \pm \sqrt{28}}{2} \\ x &= \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7} \end{aligned}$$

$$\begin{aligned} 23.) x^3 - 3x^2 - x + 3 &= 0 \\ (x^3 - 3x^2) - (x + 3) &= 0 \\ x^2(x-3) - (x+3) &= 0 \\ (x^2-1)(x-3) &= 0 \\ (x-1)(x+1)(x-3) &= 0 \end{aligned}$$

$$x=1, x=-1, x=3$$