2.5 Polynomial and Rational Functions

Fundamental Theorem of Algebra:

Examples:

- 1st degree polynomial, like $x-2$ has exactly one factor, $x-2$, and exactly 1 zero, $x=2$
- 2nd degree polynomial, like x^2-6x+9 has exactly 2 factors, $(x-3)(x-3)$, and exactly 2 zeros, $x=3\ and\ x=3$ (these are called repeating zeros)
- 3rd degree polynomial, like x^3+4x has exactly 3 factors, $x(x^2+4)=x(x-2i)(x+2i)$, and exactly 3 zeros, $x=0, x=\pm 2i$
- 4th degree polynomial, like x^4-1 has exactly 4 factors, $(x-1)(x+1)(x+i)(x-i)$ and exactly 4 zeros, $x=\pm 1,\ \ x=\pm i$

Real zeros are numbers that exist, like 1, -2 where as complex zeros aren't real because they don't exist, like $2i, -6i$

Rational Zeros Test:

Note that if the leading coefficient is 1, the possible rational zeros are simply the factors of the constant term, because if q = 1, the denominator for all possibilities is 1.

These are not the rational zeros! They are possibilities and need to be tested. We will get to testing them later.

Remember what it means to be a rational zero-one important aspect is that it must be a real number. A polynomial can have other zeros that aren't rational, but instead are complex. These're still zeros, however are not rational zeros.

Refer to example a in notes.

If there's a small amount of possible rational zeros, we can simply plug them into the function. If in doing so the function simplifies to zero, then the number plugged in is in fact a zero of the function.

Refer to example b in notes.

We can test these rational zeros by using ***synthetic division***. If we can find one rational zero, we in turn break down what is left of the polynomial to be factored, and things become much easier to factor, in turn finding the zeros.

Refer to example c, d and e in notes.

Complex Zeros Occur in Conjugate Pairs:

Refer to examples f, g, h and i in notes.

Although we know from the Fundamental Theorem of Algebra that a function of degree n has n zeros, it does not tell us how many of them are real zeros. That's where *Descarte's Rule of Signs* comes into play.

Descarte's Rule of Signs: For a polynomial of degree n with real coefficients,

Refer to example j in notes.