Chapter 7 – Gravitation

7.1 Planetary Motion and Gravitation

Nicholas Copernicus
- published *On the Revolutions of the Celestial Spheres* (1543)
- first modern astronomer to suggest the heliocentric view of the solar system

Tycho Brahe
- carefully recorded the exact positions of the planets
- concluded that the Sun and the Moon orbit Earth, and other planets orbit the Sun

Johannes Kepler
- analyzed 30 years worth of Brahe’s data
- placed the Sun at the center of the solar system
- discovered the laws that govern the motion of every planet and satellite
Kepler’s Laws

Kepler’s First Law – the paths of the planets are ellipses, with the Sun at one focus.

Kepler’s Second Law – an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals.
7.1 Planetary Motion and Gravitation

(b) The Harmony of the Worlds

Although an excellent mathematician, Kepler was also a mystic, and he indulged freely in wild speculation and the occult. In his endeavor to find an underlying harmony in nature, he constantly searched for numerological relations in the celestial realm. It was a great personal triumph, therefore, that he found a simple algebraic relation between the lengths of the semimajor axes of the planets’ orbits and their sidereal periods. Because planetary

Kepler’s Laws

Kepler’s Third Law – the square of the ratio of the periods of any two planets revolving about the Sun is equal to the cube of the ratio of their average distances from the Sun

\[ \left( \frac{T_A}{T_B} \right)^2 = \left( \frac{r_A}{r_B} \right)^3 \]

- this law relates the motion of several objects around a single body
- the other two laws can apply to a planet, moon, or satellite individually

Galileo measured the orbital sizes of Jupiter’s moons using the diameter of Jupiter as a unit of measure. He found that Io, the closest moon to Jupiter, had a period of 1.8 days and was 4.2 units from the center of Jupiter. Callisto, the fourth moon from Jupiter, had a period of 16.7 days. Using the same units that Galileo used, predict Callisto’s distance from Jupiter.

Isaac Newton

- wondered if the force acting on an apple also acts on the Moon, or beyond

Newton’s Law of Universal Gravitation

Newton’s Law of Universal Gravitation

gravitational force (F) – the force of attraction between two objects

the gravitational force is directly proportional to \( m_1 \) and \( m_2 \)

the gravitational force is inversely proportional to \( r^2 \) (inverse square law)
7.1 Planetary Motion and Gravitation

Newton related the law of universal gravitation to Kepler’s third law:

For a planet orbiting the sun: \( F_{\text{grav}} = \frac{m v^2}{r} \) and \( v = \frac{2\pi r}{T} \)

So, \( F_{\text{grav}} = \frac{m \cdot 4\pi^2 r}{T^2} \)

The force on the planet is due to the gravitational force between it and the Sun:

\[
G \frac{m_1 m_2}{r^2} = \frac{m_2 \cdot 4\pi^2 r}{T^2}
\]

Solving for \( T^2 \) \( T^2 = \left( \frac{4\pi^2}{Gm_1} \right) r^3 \)

For all planets orbiting the Sun \( \frac{r^2}{T^2} \) is constant (Kepler’s 3\(^{\text{rd}}\) Law)

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7.1 Planetary Motion and Gravitation

Solving for \( T \) in the previous equation:

\[
T = 2\pi \sqrt{\frac{r^3}{Gm_1}}
\]

Period of a Planet Orbiting the Sun

\( r \) = orbital radius
\( m_1 \) = mass of the Sun

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7.1 Planetary Motion and Gravitation

Measuring the Universal Gravitational Constant (G)

**Cavendish Experiment**

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7.1 Planetary Motion and Gravitation

Determining the mass of the Earth:

The weight of an object is a measure of Earth’s gravitational attraction

\( F_g = mg \)

That force of attraction can also be represented as \( F_g = G \frac{m_1 m_2}{r^2} \)

Setting the two values equal \( F_g = G \frac{m_1 m_2}{r^2} \)

Solving for the mass of the Earth \( m_1 \)

\[
m_1 = \frac{G r^2}{F_g} = \frac{G (9.80 \text{ m/s}^2) (6.38 \times 10^7 \text{ m})^2}{6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2} = 5.98 \times 10^{24} \text{ kg}
\]
At which point in the orbit is the Earth moving the fastest?

If $t_1 = t_2$, how does Area$_1$ compare to Area$_2$?

Having recently completed a first Physics course, a student has devised a new business plan. He learned that objects weigh different amounts at different distances from Earth's center. His plan involves buying gold by the weight at one altitude and then selling it at another altitude at the same price per weight. Should he buy at a high altitude and sell at a low altitude or vice versa?

An imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals. This is a statement of:

A. Kepler’s first law
B. Kepler’s second law
C. Kepler’s third law
D. Cavendish’s experiment

Do the data for Mercury and Jupiter agree with Kepler’s 3rd Law?

The newly discovered Kuiper Belt object, Quaoar, revolves around the Sun at a distance of about $7.5 \times 10^{12}$ m. What is Quaoar’s period?

The Moon revolves around Earth with a period of 27.32 days. Using Kepler’s third law, calculate its distance from Earth?
7.2 Using the Law of Universal Gravitation

A satellite in orbit experiences a centripetal force: 
\[ F_{\text{cent}} = \frac{m_{\text{satellite}} v^2}{r} \]

Speed of a Satellite Orbiting the Earth
\[ v = \sqrt{\frac{Gm_E}{r}} \]

Not dependent on the mass of the satellite

\( r \) = orbital radius (radius of Earth + height of satellite)
\( m_E \) = mass of the Earth

Have to be about 150 km above Earth to avoid air resistance.

Period of a Satellite Orbiting the Earth
\[ T = \frac{2\pi}{Gm_E} \sqrt{\frac{r^3}{m_E}} \]

\( r \) = orbital radius
\( m_E \) = mass of the Earth

(same equation as for the Earth orbiting the Sun)

Section 7.2

The Gravitational Field

- Gravity acts over a distance.
- In the 19th century, Michael Faraday developed the concept of a field to explain how a magnet attracts objects. Later, the field concept was applied to gravity.
- Any object with mass is surrounded by a gravitational field, in which another object experiences a force due to the interaction between its mass and the gravitational field.
- A gravitational field is denoted \( g \)

The Gravitational Field

- Gravitation is expressed by the following equation:
  \[ g = \frac{GM}{r^2} \]
- To find the gravitational field caused by more than one object, you would calculate both gravitational fields and add them as vectors.
- The gravitational field can be measured by placing an object with a small mass, \( m \), in the gravitational field and measuring the force, \( F \), on it.
- The gravitational field can be calculated using \( g = F/m \).
- The gravitational field is measured in N/kg, which is also equal to m/s².

Weight and Weightlessness

- Astronauts in a space shuttle are in an environment often called “zero-g” or “weightlessness.”
- The shuttle orbits about 400 km above Earth’s surface. At that distance, \( g \approx 8.7 \) m/s², only slightly less than on Earth’s surface. Thus, Earth’s gravitational force is certainly not zero in the shuttle.
7.2 Weight and Weightlessness

- You sense weight when something, such as the floor, or your chair, exerts a contact force on you. But if you, your chair, and the floor all are accelerating toward Earth together, then no contact forces are exerted on you.
- Thus, your apparent weight is zero and you experience weightlessness. Similarly, the astronauts experience weightlessness as the shuttle and everything in it tails freely toward Earth.

The gravitational field depends on Earth’s mass, but not on the mass of the object experiencing it.

7.2 Using the Law of Universal Gravitation

Acceleration due to gravity:

For a freely falling object \( F = G \frac{m_e m}{r^2} = m_a \)

So, \( a = G \frac{m_e}{r^2} \) we found before that \( m_e = \frac{G}{g} \)

\( a = \left( \frac{G}{r^2} \right) \frac{G}{g} \)

(This tells us that the acceleration due to gravity an object has varies with its distance from Earth)

7.2 Two Kinds of Mass

- Mass related to the inertia of an object is called inertial mass.
  \[ m_{\text{inert}} = m \]

- Mass as used in the law of universal gravitation determines the size of the gravitational force between two objects and is called gravitational mass.
  \[ m_{\text{grav}} = \frac{r^2 F_{\text{grav}}}{G} \]

Newton made the claim that inertial mass and gravitational mass are equal in magnitude. This hypothesis is called the Principle of Equivalence. All experiments conducted so far have yielded data that support this principle. Albert Einstein also was intrigued by the principle of equivalence and made it a central point in his theory of gravity.
7.2 Using the Law of Universal Gravitation

Einstein’s Theory of Gravity:

Gravity is not a force, but an effect of space itself.

Mass changes the space around it.

Mass causes space to be curved, and other bodies are accelerated because of the way they follow this curved space.

Einstein’s theory predicts the deflection or bending of light by massive objects.

Light follows the curvature of space around the massive object and is deflected.

7.2 Using the Law of Universal Gravitation

The first part of this time-lapse video shows how stars near the very center of the galaxy moved from 1992 through 1998. The area shown in these infrared images is about 0.12 parsec (0.38 light-years, or 24,600 AU) on a side. The second part of this video zooms in on the motion of one particular star called S2, and shows how this star is expected to move through 2006. The stars’ motions indicate that they are orbiting around an unseen object at the position marked by the yellow cross. Using Newton’s form of Kepler’s third law, astronomers calculate that the mass of this object is about 3.7 million solar masses. This compact, invisible object is almost certainly a supermassive black hole.

The period of a satellite orbiting Earth depends upon ________.

A. the mass of the satellite
B. the speed at which it is launched
C. the value of the acceleration due to gravity
D. the mass of Earth
7.2 Using the Law of Universal Gravitation

Your apparent weight ______ as you move away from Earth’s center.

A. decreases
B. increases
C. becomes zero
D. does not change

7.1, 7.2 Vocabulary

- Kepler’s first law
- Kepler’s second law
- Kepler’s third law
- aphelion
- perihelion
- gravitational force
- law of universal gravitation

7.1 Planetary Motion and Gravitation

Notes written by Kepler, representing the music ‘sung’ by the planets

Kepler considered the changing angular velocities when assigning the planets their harmonic proportions, and even assigned deeper meaning to his findings:

“The Earth sings Mi, Fa, Mi; you may infer even from the syllables that in this our home melody and harmony hold sway.”

7.2 Using the Law of Universal Gravitation

The Gravitational Field: another object experiences a force due to the interaction between its mass and the gravitational field, \( g \), at its location

\[
g = \frac{GM}{r^2}
\]

M = mass of the object
r = distance between object centers

Gravitational Field