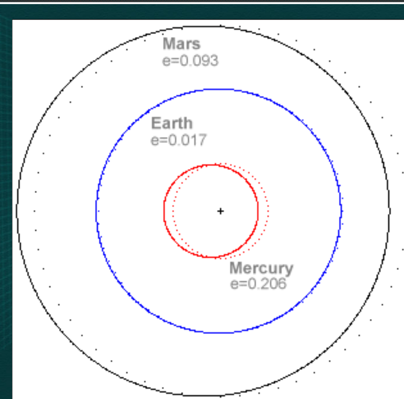
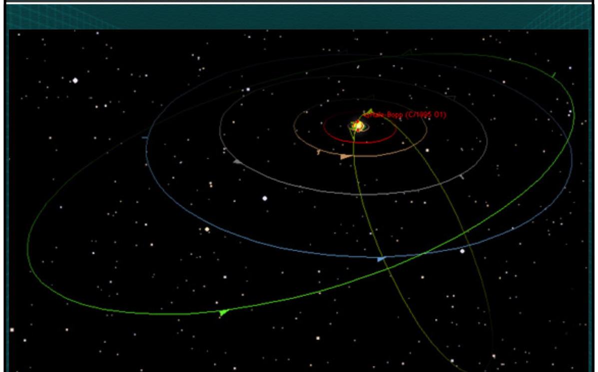
Chapter 7 – Gravitation

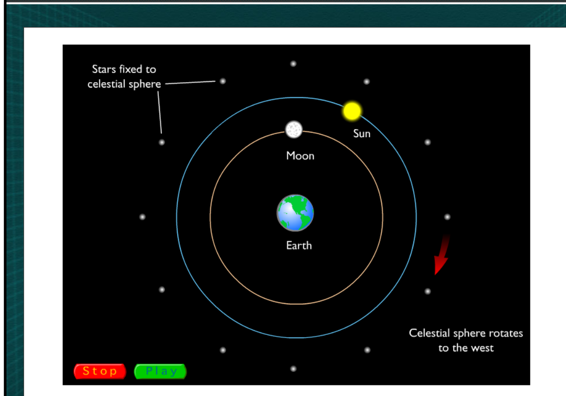
7.1 Planetary Motion and Gravitation



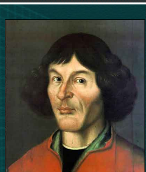
7.1 Planetary Motion and Gravitation



7.1 Planetary Motion and Gravitation



7.1 Planetary Motion and Gravitation



Nicholas Copernicus

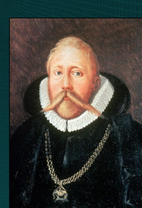
- published *On the Revolutions of the Celestial Spheres* (1543)

- first modern astronomer to suggest the heliocentric view of the solar system

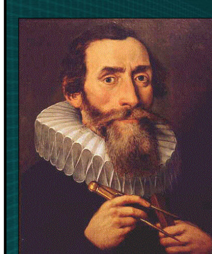
Tycho Brahe

- carefully recorded the exact positions of the planets

- concluded that the Sun and the Moon orbit Earth, and other planets orbit the Sun



7.1 Planetary Motion and Gravitation



Johannes Kepler

- analyzed 30 years worth of Brahe's data

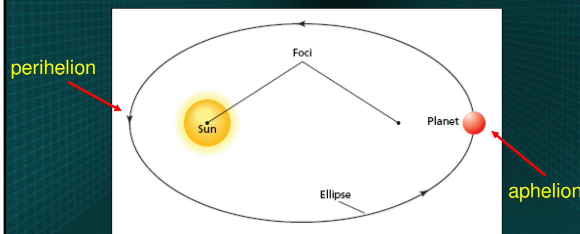
- placed the Sun at the center of the solar system

- discovered the laws that govern the motion of every planet and satellite

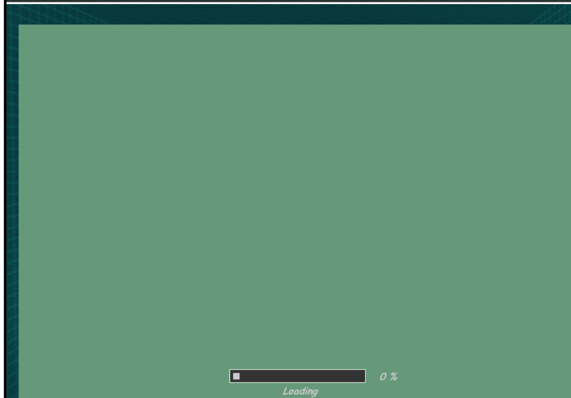
7.1 Planetary Motion and Gravitation

Kepler's Laws

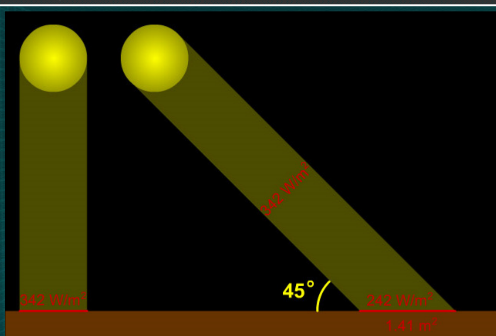
Kepler's First Law – the paths of the planets are ellipses, with the Sun at one focus



7.1 Planetary Motion and Gravitation



7.1 Planetary Motion and Gravitation

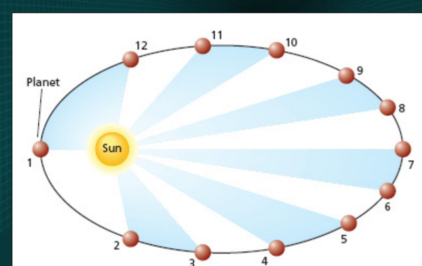


When the Sun is directly overhead, its rays strike Earth perpendicular to the ground and so deliver the maximum amount of energy. When the Sun is lower in the sky, a sunbeam strikes the ground at an angle (in the example above, 45°) and so its energy is "spread out" over a larger area... thus "diluting" its energy. In this example, the energy is spread over an area of 1.41 square meters (instead of 1 square meter when the Sun is directly overhead), so the energy per unit area is reduced from 342 W/m^2 to 242 W/m^2 (342 ÷ 1.41 = 242).

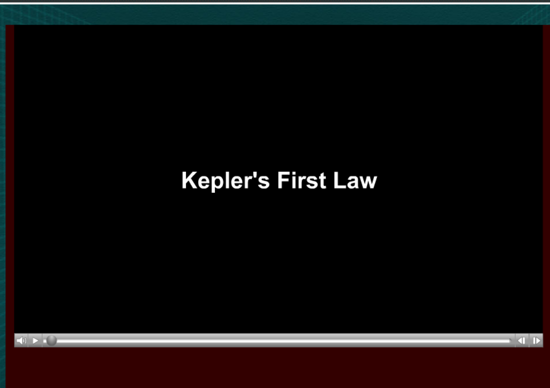
7.1 Planetary Motion and Gravitation

Kepler's Laws

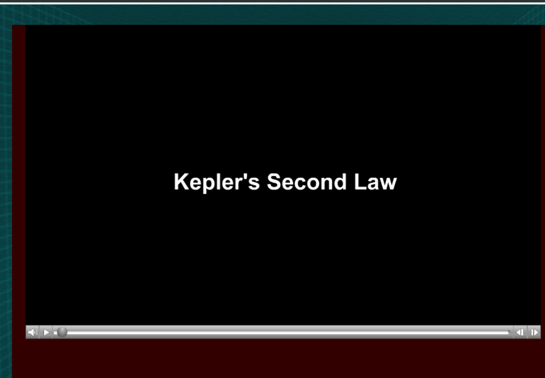
Kepler's Second Law – an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals



7.1 Planetary Motion and Gravitation



7.1 Planetary Motion and Gravitation



(b) The Harmony of the Worlds

Although an excellent mathematician, Kepler was also a mystic, and he indulged freely in wild speculation and the occult. In his endeavor to find an underlying harmony in nature, he constantly searched for numerological relations in the celestial realm. It was a great personal triumph, therefore, that he found a simple algebraic relation between the lengths of the semimajor axes of the planets' orbits and their sidereal periods. Because planetary

Data Used by Kepler (1618)

Planet	Mean distance* R (AU)	Period T (days)	R^3/T^2 $10^{-6}(AU)^3/(day)^2$
Mercury	0.389	87.77	7.64
Venus	0.724	224.70	7.52
Earth	1.000	365.25	7.50
Mars	1.524	686.98	7.50
Jupiter	5.200	4,332.62	7.49
Saturn	9.510	10,759.20	7.43

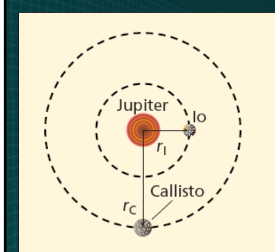
R^3 / T^2 is some constant (no matter which planet)

Kepler's Laws

Kepler's Third Law – the square of the ratio of the periods of any two planets revolving about the Sun is equal to the cube of the ratio of their average distances from the Sun

$$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$$

- this law relates the motion of several objects around a single body
- the other two laws can apply to a planet, moon, or satellite individually



Galileo measured the orbital sizes of Jupiter's moons using the diameter of Jupiter as a unit of measure. He found that Io, the closest moon to Jupiter, had a period of 1.8 days and was 4.2 units from the center of Jupiter. Callisto, the fourth moon from Jupiter, had a period of 16.7 days.

Using the same units that Galileo used, predict Callisto's distance from Jupiter.

Isaac Newton

- wondered if the force acting on an apple also acts on the Moon, or beyond

Newton's Law of Universal Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

F = magnitude of the force on a object

m_1 and m_2 = the masses of two objects

r = distance between the centers of the objects

G = universal gravitational constant (6.67×10^{-11}) $N \cdot m^2 / kg^2$

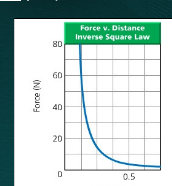
Newton's Law of Universal Gravitation $F = G\frac{m_1 m_2}{r^2}$

gravitational force (F) – the force of attraction between two objects

the gravitational force is directly proportional to m_1 and m_2

the gravitational force is inversely proportional to r^2

(inverse square law)



7.1 Planetary Motion and Gravitation

Newton related the law of universal gravitation to Kepler's third law:

For a planet orbiting the sun: $F_{net} = \frac{m_p v^2}{r}$ and $\left(v = \frac{2\pi r}{T}\right)$

So, $F_{net} = \frac{m_p 4\pi^2 r}{T^2}$

The force on the planet is due to the gravitational force between it and the Sun:

$$G\frac{m_s m_p}{r^2} = \frac{m_p 4\pi^2 r}{T^2}$$

Solving for T^2: $T^2 = \left(\frac{4\pi^2}{Gm_s}\right) r^3$

For all planets orbiting the Sun $\frac{T^2}{r^3}$ is constant (Kepler's 3rd Law)

7.1 Planetary Motion and Gravitation

Solving for T in the previous equation: $T^2 = \left(\frac{4\pi^2}{Gm_s}\right) r^3$

Period of a Planet Orbiting the Sun

$$T = 2\pi\sqrt{\frac{r^3}{Gm_s}}$$

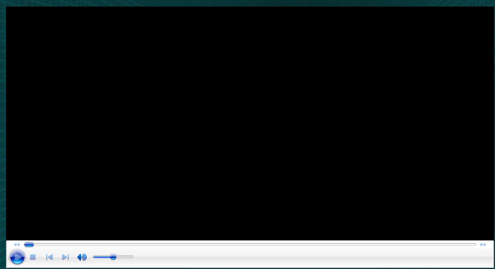
r = orbital radius

m_s = mass of the Sun

7.1 Planetary Motion and Gravitation

Measuring the Universal Gravitational Constant (G)

7.1 Planetary Motion and Gravitation



7.1 Planetary Motion and Gravitation

Determining the mass of the Earth:

The weight of an object is a measure of Earth's gravitational attraction

$$F_g = mg$$

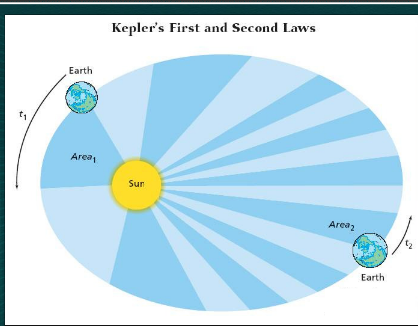
That force of attraction can also be represented as $F_g = G\frac{m_E m}{r_E^2}$

Setting the two values equal $\cancel{m}g = G\frac{m_E \cancel{m}}{r_E^2}$

Solving for the mass of the Earth $m_E = \frac{g r_E^2}{G}$

$$m_E = \frac{(9.80\,m/s^2)(6.38\,x\,10^6\,m)^2}{(6.67\,x\,10^{-11}\,N\cdot m^2/kg^2)} = 5.98\,x\,10^{24}\,kg$$

7.1 Planetary Motion and Gravitation



At which point in the orbit is the Earth moving the fastest?

If $t_1 = t_2$, how does $Area_1$ compare to $Area_2$?

7.1 Planetary Motion and Gravitation

Planet	Minium Distance from Sun (km)	Maxium Distance from Sun (km)	Average Distance from Sun (km)	Period (Earth Years)
Mercury	4.608×10^7	6.982×10^7	5.791×10^7	0.241
Venus	1.075×10^8	1.089×10^8	1.082×10^8	0.615
Earth	1.471×10^8	1.521×10^8	1.496×10^8	1.000
Mars	2.066×10^8	2.492×10^8	2.279×10^8	1.881
Jupiter	7.405×10^8	8.166×10^8	7.786×10^8	11.860
Saturn	1.353×10^9	1.515×10^9	1.434×10^9	29.420
Uranus	2.741×10^9	3.004×10^9	2.872×10^9	84.010
Neptune	4.444×10^9	4.546×10^9	4.495×10^9	164.790
Pluto	4.435×10^9	7.304×10^9	5.870×10^9	247.680

Do the data for Mercury and Jupiter agree with Kepler's 3rd Law?

The newly discovered Kuiper Belt object, Quaoar, revolves around the Sun at a distance of about 7.5×10^{12} m. What is Quaoar's period?

The Moon revolves around Earth with a period of 27.32 days. Using Kepler's third law, calculate its distance from Earth?

7.1 Planetary Motion and Gravitation

Having recently completed a first Physics course, a student has devised a new business plan. He learned that objects weigh different amounts at different distances from Earth's center. His plan involves buying gold by the weight at one altitude and then selling it at another altitude at the same price per weight. Should he buy at a high altitude and sell at a low altitude or vice versa?

7.1 Planetary Motion and Gravitation

Which of the following is true according to Kepler's first law?

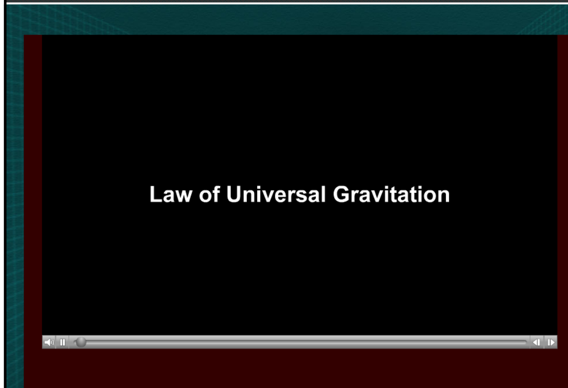
A. Paths of planets are ellipses with Sun at one focus.
B. Any object with mass has a field around it.
C. There is a force of attraction between two objects.
D. Force between two objects is proportional to their masses.

7.1 Planetary Motion and Gravitation

An imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals. This is a statement of:

A. Kepler's first law
B. Kepler's second law
C. Kepler's third law
D. Cavendish's experiment

7.2 Using the Law of Universal Gravitation



7.2 Using the Law of Universal Gravitation

A satellite in orbit experiences a centripetal force $F_{net} = \frac{m_{sat} v^2}{r}$

The F_{net} equals $G\frac{m_{sat} m_E}{r^2}$

$$G\frac{\not{m}_{sat} m_E}{r^2} = \frac{\not{m}_{sat} v^2}{r}$$

Speed of a Satellite Orbiting the Earth

$$v = \sqrt{\frac{Gm_E}{r}}$$

Not dependent on the mass of the satellite

r = orbital radius (radius of Earth + height of satellite)

m_E = mass of the Earth

Have to be about 150 km above Earth to avoid air resistance

7.2 Using the Law of Universal Gravitation

Period of a Satellite Orbiting the Earth

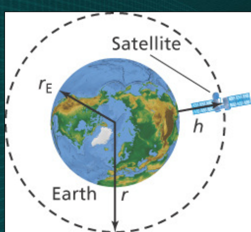
$$T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$$

r = orbital radius

m_E = mass of the Earth

(same equation as for the Earth orbiting the Sun)

7.2 Using the Law of Universal Gravitation

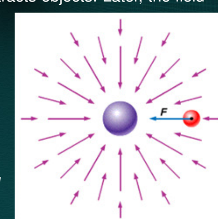


Assume that a satellite orbits Earth 225 km above its surface. Given that the mass of Earth is 5.97 × 10^{24} kg and the radius of Earth is 6.38 × 10^6 m, what are the satellite's orbital speed and period?

Section 7.2

The Gravitational Field

- Gravity acts over a distance.
- (In the 19th century, Michael Faraday developed the concept of a field to explain how a magnet attracts objects. Later, the field concept was applied to gravity.)
- Any object with mass is surrounded by a **gravitational field** in which another object experiences a force due to the interaction between its mass and the gravitational field
- A gravitational field is denoted ***g***



Section 7.2

The Gravitational Field

- Gravitation is expressed by the following equation:

$$g = \frac{GM}{r^2}$$

- To find the gravitational field caused by more than one object, you would calculate both gravitational fields and add them as vectors.
- The gravitational field can be measured by placing an object with a small mass, *m*, in the gravitational field and measuring the force, *F*, on it.
- The gravitational field can be calculated using $g = F/m$.
- The gravitational field is measured in N/kg, which is also equal to m/s².

Section 7.2

Weight and Weightlessness

- Astronauts in a space shuttle are in an environment often called "zero-*g*" or "weightlessness."
- The shuttle orbits about 400 km above Earth's surface. At that distance, g = 8.7 m/s², only slightly less than on Earth's surface. Thus, Earth's gravitational force is certainly not zero in the shuttle.

Section 7.2

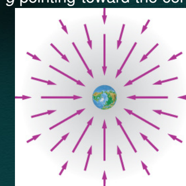
Weight and Weightlessness

- You sense weight when something, such as the floor, or your chair, exerts a contact force on you. But if you, your chair, and the floor all are accelerating toward Earth together, then no contact forces are exerted on you.
- Thus, your apparent weight is zero and you experience weightlessness. Similarly, the astronauts experience weightlessness as the shuttle and everything in it falls freely toward Earth.

Section 7.2 Using the Law of Universal Gravitation

The Gravitational Field

- On Earth's surface, the strength of the gravitational field is 9.80 N/kg, and its direction is toward Earth's center. The field can be represented by a vector of length *g* pointing toward the center of the object producing the field.
- The strength of the field varies inversely with the square of the distance from the center of Earth.
- The gravitational field depends on Earth's mass, but not on the mass of the object experiencing it.



(You can picture the gravitational field of Earth as a collection of vectors surrounding Earth and pointing toward it, as shown in the figure.)

7.2 Using the Law of Universal Gravitation

Acceleration due to gravity:

For a freely falling object $F = G\frac{m_E m}{r^2} = ma$

So, $a = G\frac{m_E}{r^2}$ we found before that $m_E = \frac{g r_E^2}{G}$

$$a = g\left(\frac{r_E}{r}\right)^2$$

(this tells us that the acceleration due to gravity an object has varies with its distance from Earth)

Section 7.2 Using the Law of Universal Gravitation

Two Kinds of Mass

- Mass related to the inertia of an object is called **inertial mass**.

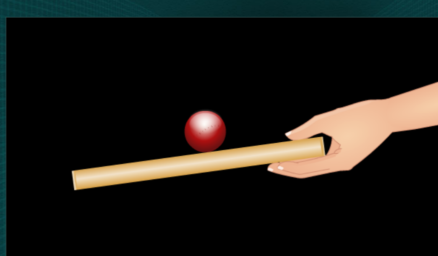
$$m_{inertial} = \frac{F_{net}}{a}$$

- Mass as used in the law of universal gravitation determines the size of the gravitational force between two objects and is called **gravitational mass**.

$$m_{grav} = \frac{r^2 F_{grav}}{Gm}$$

Section 7.2 Using the Law of Universal Gravitation

Two Kinds of Mass



Click image to view the movie.

Section 7.2 Using the Law of Universal Gravitation

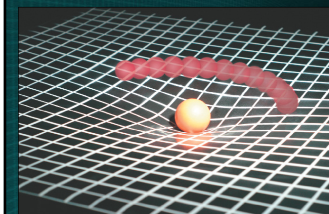
Two Kinds of Mass

- Newton made the claim that inertial mass and gravitational mass are equal in magnitude. This hypothesis is called the *Principle Of Equivalence*. All experiments conducted so far have yielded data that support this principle. Albert Einstein also was intrigued by the principle of equivalence and made it a central point in his theory of gravity.

Resources End

7.2 Using the Law of Universal Gravitation

Einstein's Theory of Gravity:

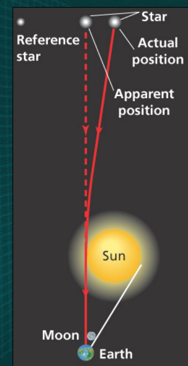


Gravity is not a force, but an effect of space itself.

Mass changes the space around it.

Mass causes space to be curved, and other bodies are accelerated because of the way they follow this curved space.

7.2 Using the Law of Universal Gravitation



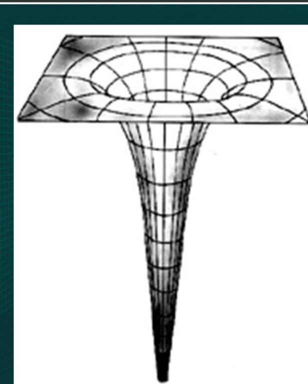
Einstein's theory predicts the deflection or bending of light by massive objects.

Light follows the curvature of space around the massive object and is deflected.

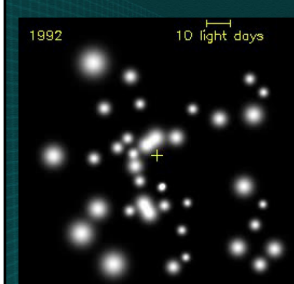
7.2 Using the Law of Universal Gravitation

In the formation known as Einstein's Cross four images of the same distant quasar appears around a foreground galaxy due to strong gravitational lensing

7.2 Using the Law of Universal Gravitation



7.2 Using the Law of Universal Gravitation



The first part of this time-lapse video shows how stars near the very center of the galaxy moved from 1992 through 1998. The area shown in these infrared images is about 0.12 parsec (0.38 light-years, or 24,000 AU) on a side. The second part of the video zooms in on the motion of one particular star called S2, and shows how this star is expected to move through 2006.

The stars' motions indicate that they are orbiting around an unseen object at the position marked by the yellow cross. Using Newton's form of Kepler's third law, astronomers calculate that the mass of this object is about 3.7 million solar masses. This compact, invisible object is almost certainly a supermassive black hole.

7.2 Using the Law of Universal Gravitation

The period of a satellite orbiting Earth depends upon __________.

A. the mass of the satellite

B. the speed at which it is launched

C. the value of the acceleration due to gravity

D. the mass of Earth

7.2 Using the Law of Universal Gravitation

Your apparent weight __________ as you move away from Earth's center.

A. decreases

B. increases

C. becomes zero

D. does not change

7.1, 7.2 Vocabulary

Kepler's first law

Kepler's second law

Kepler's third law

aphelion

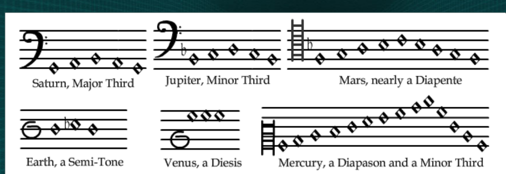
perihelion

gravitational force

law of universal gravitation

a)Copernicus b) Kepler c) Brahe d) Newton

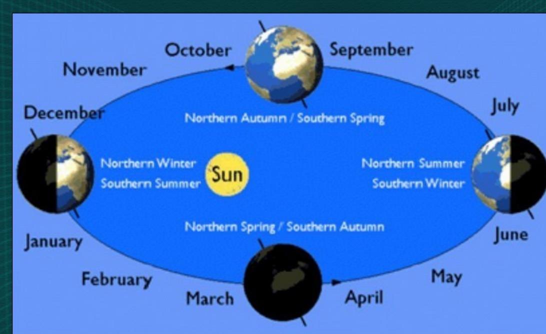
7.1 Planetary Motion and Gravitation



Notes written by Kepler, representing the music "sung" by the planets

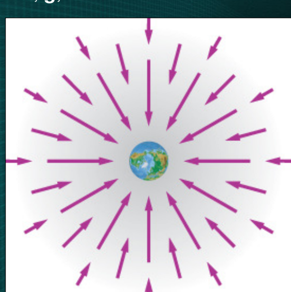
Kepler considered the changing angular velocities when assigning the planets their harmonic proportions, and even assigned deeper meaning to his findings: "The Earth sings Mi, Fa, Mi: you may infer even from the syllables that in this our home **mi**sery and **fa**mine hold sway.[9]"

7.1 Planetary Motion and Gravitation



7.2 Using the Law of Universal Gravitation

The Gravitational Field: another object experiences a force due to the interaction between its mass and the gravitational field, **g**, at its location



Gravitational Field

$$g = \frac{GM}{r^2}$$

M = mass of the object

r = distance between object centers

7.2 Using the Law of Universal Gravitation

The Gravitational Field: another object experiences a force due to the interaction between its mass and the gravitational field, **g**, at its location

Gravitational Field

$$g = \frac{GM}{r^2}$$

M = mass of the object (w/ field)

r = distance between object centers