Solutions to Sequence and Series Review
Chapter 9 Sections 1 – 4

Section 1:

1a) $a_1 = -\frac{1}{2}$, $a_2 = \frac{2}{3}$, $a_3 = -\frac{3}{4}$, $a_4 = \frac{4}{5}$, $a_5 = -\frac{5}{6}$

1b) $a_1 = -2$, $a_2 = 0$, $a_3 = \frac{2}{3}$, $a_4 = 1$, $a_5 = \frac{6}{5}$

2a) $a_{13} = \frac{52}{7}$

3a) $a_n = \frac{n+2}{2^n}$

3b) $a_n = (-1)^n(4n)$

4a) 4320

5a) 84

5b) $-\frac{117}{161} \approx -0.727$

5c) $\frac{1}{3}$

Section 2:

1a) Yes; d = -3

1b) Not Arithmetic

2a) $a_1 = -3$, $a_2 = -8$, $a_3 = -13$, $a_4 = -18$, $a_5 = -23$

3a) $a_n = 4n + 3$

3b) $a_n = \frac{1}{2}n + \frac{9}{2}$

3c) $a_n = -15n + 265$

4) $a_{21} = 203$

5a) $S_{12} = 282$

5b) $S_{30} = 2790$

6a) Salary on the 10^{th} year is $55,500

6b) Total earned over is $255,000

Section 3:

1a) Yes; r = 3

1b) Not Geometric

2a)
$$a_1 = 6$$
$$a_2 = 3$$
$$a_3 = \frac{3}{2} = 1.5$$
$$a_4 = \frac{3}{4} = 0.75$$
$$a_5 = \frac{3}{8} = 0.375$$

3a) $a_n = 5(3)^{n-1}$

$a_{12} = 885{,}735$

3b) $a_n = 3(4)^{n-1}$

$a_{10} = 786{,}432$

4a) $\frac{4275}{256} \approx 16.7$

5a) 3.75

6) $29,562.84

Section 4:

1a) $2^3 + 4^3 + 6^3 + \cdots + (2n)^3 = 2n^2(n+1)^2$

1. Show S_1 is true. $2(1)^2(1+1)^2 = 2(4) = 8$ (note $2^3 = 8$)
2. Assume $S_k = 2^3 + 4^3 + 6^3 + \cdots + (2k)^3 = 2k^2(k+1)^2$
3. Show $S_{k+1} = 2(k+1)^2(k+2)^2$
4. $S_{k+1} = 2^3 + 4^3 + 6^3 + \cdots + (2k)^3 + (2(k+1))^3$

$$= 2k^2(k+1)^2 + (2(k+1))^3$$
$$= 2(k+1)^2\left[k^2 + 2^2(k+1)\right]$$
$$= 2(k+1)^2\left[k^2 + 4k + 4\right]$$
$$= 2(k+1)^2(k+2)^2$$

$\therefore$ $2^3 + 4^3 + 6^3 + \cdots + (2n)^3 = 2n^2(n+1)^2$ is valid for all positive integers n.

2a) 29,280