3.2 Continued: Graphing Logarithmic and Natural Logarithmic Functions

Graphing Logarithmic Functions $y = \log_a x$

It is easier to understand these properties/characteristics of the graphs of a logarithm by remembering:

$y = \log_a x$ IS EQUIVALENT TO $x = a^y$

Ex): $y = \log_2 x$ IS EQUIVALENT TO $x = 2^y$

Transformations of Graphs of Logarithmic Functions

horizontal shifts

$\log_a(x-h)$ graph shifts to the right
$\log_a(x+h)$ graph shifts to the left

veritcal shifts

$\log_a(x)+k$ graph shifts up
$\log_a(x)-k$ graph shifts down

Axis flips

$\log_a(-x)$ graph flips over the y axis
$-\log_a(x)$ graph flips over the x axis

Domain:

- ***If there is no horizontal shift***, the domain is $(0,\infty)$ → the domain of a function consists of all x values for which a function is defined/exists, and excludes all x values that make the function undefined. The domain of a logarithmic function is the set of all positive real numbers because no matter what, x will never be negative (refer to the exponential form of a log above, and try to plug in values for y. There is no y value that will yield a negative x value.) So, the domain are all values for which $x > 0$, which is $(0,\infty)$
- ***if there is a horizontal shift***, the domain is $(-c,\infty)$
    - Ex a): Find the domain of $y = \log_2(x+4)$

- ***if there is a horizontal shift and a slip over the y axis*** the domain is $(-\infty,-c)$
    - Ex b): Find the domain of $y = \log_2(-x+3)$

Range: ALWAYS $(-\infty,\infty)$

x-intercept:

- ***If there are no shifts*** the x intercept is $(1,0)$ → this is because the x-intercept is found by setting y = to 0, and solving for x. If we do this to the parent function, $y = \log_a x$ we would get $0 = \log_a x$ which is $a^0 = x$ in exponential form, and we know that ANYTHING raised to the power of 0, is 1. Therefore, x = 1
- ***If there are shifts,*** the x intercept is likely to change. As usual, to find the x intercept(s), set y=0, and solve for x. this will require you to set the logarithm = to 0 to solve. It's easiest (for now) to rewrite the equation as an exponential, making it easier to solve
    - Ex c): Find the x-intercept of $y = \log_2(x+4)$

y-intercept:

- ***No shifts:*** there are no y-intercepts NONE → this is because the y-intercept is found by plugging 0 in for x, and solving for y. If we do this for the parent function, $y = \log_a x$ we would get $y = 0$ which is $a^y = 0$ in exponential form. In this case, there is NO REAL NUMBER that y could be to make this function = 0 (there is no exponent/base combination that exists that will give you an answer of 0).
- ***If there are shifts,*** there is likely a y-intercept → as we usually do to find the y intercept(s) of a function, set x=0, and solve for y. It is easiest (for now) to rewrite this equation as an exponential, this will make it much easier to solve. If, when solving, you are unable to, or come out with an exponential that simply does not make sense, there is no y-intercept
    - Ex d): Find the y-intercept of $y = \log_2(x+4)$

Vertical asymptote:

- ***If there is no horizontal shift:*** vertical asymptote at $y = 0$ → vertical asymptotes are "imaginary lines" in which the graph of a function cannot cross through. This is because vertical asymptotes are "x equals equations," found by finding which x values will make the function not exist. If a function does not exist at certain x values, its graph CANNOT exist at those values either!!

  ** your graph will APPROACH this asymptote, but will NEVER touch/cross it**

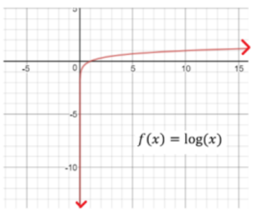
  (the exception to this is if there is a hole in the graph at that point, which you will learn about in calc, but for now, graphs of functions can NEVER go through vertical asymptotes)
- ***If there is a horizontal shift*** the vertical asymptote is at $x = -h$
    - Ex e): Find the horizontal asymptote of $y = \log_2(x+4)$

Graphs of logarithmic functions DO NOT have horizontal **asymptotes** because there is not one y that would be considered "the largest y value possible for any given x value." Think about the parent function and its equivalent exponential: $y = \log_a x$ and $x = a^y$ if x can be anything larger than 0, the larger x is, the larger y is, and because there is an infinite amount of positive numbers (possible x values), there is an infinite amount of resulting y values as well.

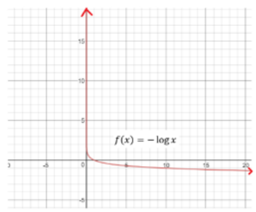
How to graph log functions (simplistically):

1.) plot all intercepts, and create any vertical asymptotes
2.) find/identify all shifts (shifts are based upon the parent graph)
3.) compare the function to the graph of its parent function to know about the "end behavior"
4.) Using info from steps 1,2 and 3, create a rough sketch of the graph

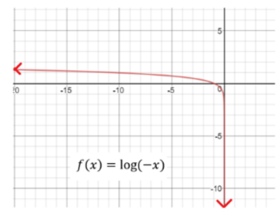
If there is **no flip over the x or y axis**, the graph will resemble the original parent function:



If there is a **flip over the x axis** (the entire function is negated, denoted with a negative in the front), the graph will resemble an x axis flip of the parent function:



If there is a **flip over the y axis** (the x value is negated), the graph will resemble a y axis flip of the parent function:



Graphing natural logarithmic functions $y = \ln(x)$

It is easier to understand these properties/characteristics of the graphs of a logarithm by remembering:

$y = \ln x$ IS EQUIVALENT TO $e^y = x$

(remember, the base of a natural log is ALWAYS the natural exponential, e, but is never written)

Ex): $y = \ln(x + 6)$ IS EQUIVALENT TO $e^y = x + 6$

- Transformations: The same as the transformations of log functions
- Domain: found the same way the domain of a log is found
  - Ex f): Find the domain of $y = \ln(x - 5)$

  $x - 5 > 0$ → $x > 5$ → domain is $(5, \infty)$ this is because this natural log is the same as $e^y = x - 5$, and as soon as x is smaller than 5, the exponential would not make sense. For ex, if $x = 4$, we would have (in exponential form) $e^y = 4 - 5$ → $e^y = -1$ → this DOES NOT MAKE SENSE because there is NO exponent that exists that will give us a NEGATIVE answer! (remember, $e \approx 2.718281828 \ldots$ which is POSITIVE. All numbers smaller than 5 in this case would give us a similar exponential equation that WOULD NOT make sense
- Range: always $(-\infty, \infty)$

- x-intercept: found the same way the x-intercept of a log is found
  - Ex g): Find the x intercepts of $y = \ln(x - 5)$

  Write as an exponential → $e^y = x - 5$ Plug 0 in for y, and solve for x → $e^0 = x - 5$ → $e^0 + 5 = x$ → $1 + 5 = x$ → $6 = x$
  X intercept is $(6, 0)$

- y-intercept: found the same way the y-intercept of a log is found

- Vertical asymptote: found the same way a vertical asymptote of a log is found. For now, we are not able to find the y intercepts (if they exist) because we do not yet know how to solve for a variable within an exponent
- natural log functions also do not have horizontal asymptotes