Chapter 3: Exponential and Logarithmic Functions

These functions are examples of transcendental functions. Until now, we have been dealing with algebraic functions.

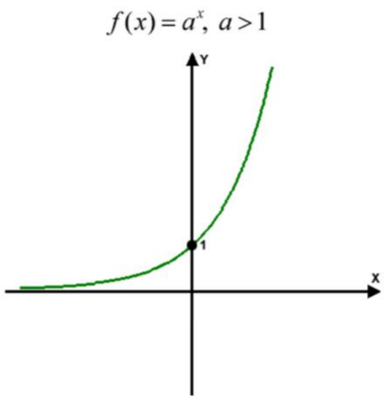
Section 1: Exponential Functions

Definition of Exponential Function

The exponential function f with base a is denoted by

$$f(x) = a^x$$

Where $a > 0$, $a \neq 1$, and x is any real number.



- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$
- Intercept: $(0,1)$
- x-axis is horizontal asymptote $\left(a^x \to 0 \quad as \quad x \to -\infty\right)$
- Continuous

Transformations of Graphs of Exponential Functions

Horizontal Translations

$y = a^{x-c}$ graph shifts to the right

$y = a^{x+c}$ graph shifts to the left

Vertical Translations

$y = a^x + c$ graph shifts up

$y = a^x - c$ graph shifts down

Axis flips

$y = a^{-x}$ graph flips over the y axis

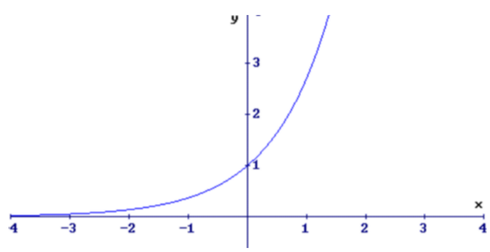
$y = -a^x$ graph flips over the x axis

One-to-One Property

For $a > 0$ and $a \neq 1$, $a^x = a^y$ if and only if $x = y$.

The Natural Base *e*

- e is an irrational number
- $e \approx 2.718281828$ …
- This number is called the *natural base*.
- The function given by $f(x) = e^x$ is called the *natural exponential function*.
- $y_1 = \left(1 + \frac{1}{x}\right)^x$ as $x \to \infty$ is the same as $y_2 = e$



Applications

Formulas for Compounded Interest

After t years, the balance A in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas.

1. For n compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. For continuous compounding: $A = Pe^{rt}$