The Birthday Problem: The Easy Way!

You may already understand this, as it's the way that's usually used. In fact, in the first 20 responses to a Google search for the Birthday Problem, it was invariably done the exact same way. Here's a quick explanation: in probability, the set of all possible outcomes of a trial is equal to the number of ways of succeeding plus the number of ways of failing and is thus equal to 1 when you divide the number of ways of succeeding + the number of ways of failing by the total number of possible outcomes. So:

Total probability for an event = Sum of all possible outcomes of a trial divided by the total number of possible outcomes in the sample space, which equals 1, since the numerator and denominator are always equal

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P(success) = p
P(failure) = 1-p = q
P(success) + P(failure) = p + q = p + 1-p = 1
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Now, sometimes it's difficult to directly calculate the probability of success--as in the birthday problem--so we can use a simple mathematical trick to figure the probability in a different way. In our case, success of our trial is defined as finding at least 2 matching birthdays in a group of n people. Failure is not finding any matching birthdays at all, in which case everyone in the group has a different birthday. This is actually quite convenient, as we will see later on in the problem, because there is only one case for failure, whereas the # of possible cases for success is rather daunting and makes the calculations prohibitively taxing and messy. More on that later :) For now, the reason we want to find the # of ways to fail is that we can subtract the probability of failure from one to get the probability of success:

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p + q = 1 (From above)
p = 1 - q (the probability of success is just the total
probability minus the probability of failure)
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Now, let's see how we can fail to find 2 people with the same birthday in a group of n people. Well, look at the first person. There are 365 birthdays that person could have in order to not match anyone in the group of 0 people we've already looked at so far. So with one person, we have 365/365 ways to fail. Now let's look at the second person. Since that person can't have the same birthday as the first person, 364 out of 365 choices of birthdays will fail to match the first person's b-day, so taken in light of the first person's choice of birthdays, we have 365/365 * 364/365 = 364/365. Now, look at the third person. That person has 363 ways to not choose either of the previous 2 people's birthdays. This gives us 365/365 * 364/365 * 363/365 = 364/365 * 363/365 ways to fail to find a pair in a group of 3. This continues until we reach n, and we have the following summation: For i from 1 to n, sum[(365-i+1)/365]. Now that we have the probability of failure, we can just subtract it from one, and we have our probability of success! That was incredibly easy compared to the following ways of trying to calculate the probability of success directly.