

3-3 Study Guide and Intervention

Rate of Change and Slope

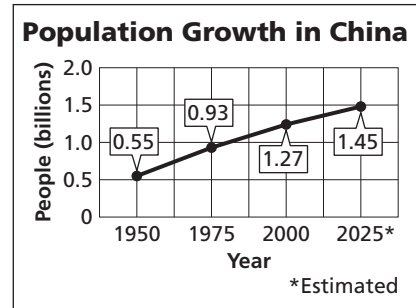
Rate of Change The **rate of change** tells, on average, how a quantity is changing over time.

Example **POPULATION** The graph shows the population growth in China.

- a. Find the rates of change for 1950–1975 and for 2000–2025.

$$\begin{aligned} 1950-1975: \frac{\text{change in population}}{\text{change in time}} &= \frac{0.93 - 0.55}{1975 - 1950} \\ &= \frac{0.38}{25} \text{ or } 0.0152 \end{aligned}$$

$$\begin{aligned} 2000-2025: \frac{\text{change in population}}{\text{change in time}} &= \frac{1.45 - 1.27}{2025 - 2000} \\ &= \frac{0.18}{25} \text{ or } 0.0072 \end{aligned}$$



Source: United Nations Population Division

- b. Explain the meaning of the rate of change in each case.

From 1950–1975, the growth was 0.0152 billion per year, or 15.2 million per year.
From 2000–2025, the growth is expected to be 0.0072 billion per year, or 7.2 million per year.

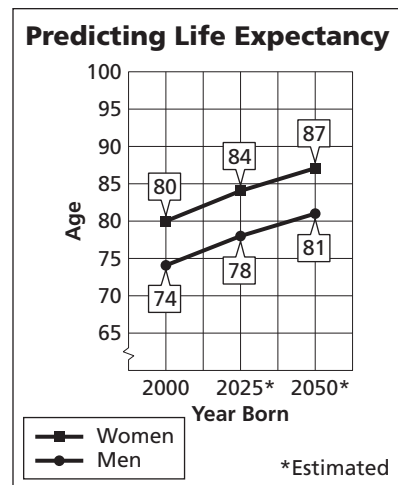
- c. How are the different rates of change shown on the graph?

There is a greater vertical change for 1950–1975 than for 2000–2025. Therefore, the section of the graph for 1950–1975 has a steeper slope.

Exercises

1. **LONGEVITY** The graph shows the predicted life expectancy for men and women born in a given year.

- Find the rates of change for women from 2000–2025 and 2025–2050.
- Find the rates of change for men from 2000–2025 and 2025–2050.
- Explain the meaning of your results in Exercises 1 and 2.
- What pattern do you see in the increase with each 25-year period?



Source: USA TODAY

- Make a prediction for the life expectancy for 2050–2075. Explain how you arrived at your prediction.

3-3 Study Guide and Intervention *(continued)***Rate of Change and Slope**

Find Slope The **slope** of a line is the ratio of change in the y -coordinates (rise) to the change in the x -coordinates (run) as you move in the positive direction.

Slope of a Line	$m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are the coordinates of any two points on a nonvertical line
------------------------	---

Example 1 Find the slope of the line that passes through $(-3, 5)$ and $(4, -2)$.

Let $(-3, 5) = (x_1, y_1)$ and $(4, -2) = (x_2, y_2)$.

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\
 &= \frac{-2 - 5}{4 - (-3)} && y_2 = -2, y_1 = 5, x_2 = 4, x_1 = -3 \\
 &= \frac{-7}{7} && \text{Simplify.} \\
 &= -1
 \end{aligned}$$

Example 2 Find the value of r so that the line through $(10, r)$ and $(3, 4)$ has a slope of $-\frac{2}{7}$.

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\
 -\frac{2}{7} &= \frac{4 - r}{3 - 10} && m = -\frac{2}{7}, y_2 = 4, y_1 = r, x_2 = 3, x_1 = 10 \\
 -\frac{2}{7} &= \frac{4 - r}{-7} && \text{Simplify.} \\
 -2(-7) &= 7(4 - r) && \text{Cross multiply.} \\
 14 &= 28 - 7r && \text{Distributive Property} \\
 -14 &= -7r && \text{Subtract 28 from each side.} \\
 2 &= r && \text{Divide each side by } -7.
 \end{aligned}$$

Exercises

Find the slope of the line that passes through each pair of points.

- $(4, 9), (1, 6)$
- $(-4, -1), (-2, -5)$
- $(-4, -1), (-4, -5)$
- $(2, 1), (8, 9)$
- $(14, -8), (7, -6)$
- $(4, -3), (8, -3)$
- $(1, -2), (6, 2)$
- $(2, 5), (6, 2)$
- $(4, 3.5), (-4, 3.5)$

Find the value of r so the line that passes through each pair of points has the given slope.

- $(6, 8), (r, -2), m = 1$
- $(-1, -3), (7, r), m = \frac{3}{4}$
- $(2, 8), (r, -4), m = -3$
- $(7, -5), (6, r), m = 0$
- $(r, 4), (7, 1), m = \frac{3}{4}$
- $(7, 5), (r, 9), m = 6$