It is VERY important that you remember PEMDAS for this exam!!!!

Simplifying Expressions with Exponents (Remember the order of operations – PEMDAS)

Ex) Simplify the expressions. a) $\frac{2}{5}[37-(2-5)]^2$ b) $3x(7x+1)^2$

Evaluating Expressions Given x and y Values (Remember the order of operations – PEMDAS)

Ex) Evaluate. a) $(x-y)^2 \div 2y - x^2 \quad if\ x=-4\ and\ y=2$ b) $3x - x^2 + (2x+y)^2 \quad if\ x=-1\ \ and\ \ y=-2$

Writing Equations Given Algebraic Sentences **Write the equation *one piece at a time* to avoid confusion!

Ex) Write an equation for the following sentences.

a) The difference of a number n and 12 is 4 less than triple the number.

b) The product of 45 and a number k is 3 more than the difference of that number and 10.

c) The quotient of a number n and 30 is 2 less than 7 less than the product of 2 and the number.

Simplifying Expressions Involving Fractions (Remember the order of operations – PEMDAS)

******When you multiply fractions, you multiply straight across- numerator $\times$ numerator and denominator $\times$ denominator

**Any number can be written as a fraction by putting it over 1 (ex: $5=\frac{5}{1}$)

Ex) Simplify the following. a) $18\left(\frac{1}{2}x+\frac{1}{3}y\right)-\frac{1}{3}(9x-6y)$ b) $20\left(\frac{1}{5}y-\frac{1}{2}x\right)+\frac{1}{4}(8x+12y)$

Simplifying Expressions Using Properties of Exponents

Property	Example
Product of Powers $a^m \bullet a^n = a^{m+n}$	$x^2 \bullet x^7 \bullet x = x^{2+7+1} = x^{10}$
Power of a Power $(a^m)^n = a^{mn}$	$(n^3)^4 = n^{3\bullet 4} = n^{12}$
Power of Product $(ab)^m = a^m b^m$	$(xy)^5 = x^5y^5$
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^{11}}{x^4} = x^{11-4} = x^7$
Power of a Quotient $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$	$(\frac{x}{y})^5 = \frac{x^5}{y^5}$
Negative Exponents $a^{-n} = \frac{1}{a^n}$ $a \neq 0$ $a^n = \frac{1}{a^{-n}}$ $a \neq 0$	$d^{-2} = \frac{1}{d^2}$ $\frac{1}{y^{-7}} = y^7$

Ex) Simplify the following: a) $\frac{x^3y^2}{5xy} \bullet (\frac{3xy}{x^{-2}y^4})^2$ b) $\frac{2x^4y^3}{x^2y} \bullet (\frac{4x^2y}{x^{-1}y^{-3}})^3$

Solving Equations

Basic Algebraic Equations: Get x by itself on one side (solving for x)

Ex) Solve for x. a) $6x + 2(x + 4) = -72$ b) $14x + 7(x + 4) = -14$

Equations with Fractions

**When you are unsure how to get x by itself when it is involved in a fraction, give yourself a simpler problem like $\frac{6}{2} = 3$ and ask yourself how you would solve to get 6 if it was x, 2 if it was x or 3 if it was x.

Ex) Solve for x. a) $\frac{x}{9} + 2 = 5$ b) $\frac{x}{11} - 1 = 7$

Equations with Absolute Value ***Make sure to check your solutions!!!***

Ex) Solve for x. a) $|2x - 4| = 16$ b) $|3x - 3| = 12$

Equations with One Fraction on Each Side of the Equals Sign – Use Cross Multiplication!

$$\frac{x+2}{3} = \frac{3x-2}{6} \quad \Rightarrow \quad \frac{x+2}{3} \times \frac{3x-2}{6}$$

1) Cross multiply: $6(x+2) = 3(3x-2)$

2) Distribute: $6x + 12 = 9x - 6$

3) Solve: $18 = 3x$

4) Final Answer: $\boxed{x = 6}$

5) Check: $\frac{(6)+2}{3} = \frac{3(6)-2}{6}$

$\frac{8}{3} = \frac{16}{6}$ ✓

Ex) Solve the equations.

a) $\frac{4}{5} = \frac{x}{30}$ b) $\frac{2x}{9x-4} = \frac{2}{11}$

Equations with Radicals

1. If x is inside of a radical, square both sides to get rid of the radical (squaring a square root cancels both operations out)
2. ALWAYS MAKE SURE YOU PLUG YOUR SOLUTIONS BACK INTO THE EQUATION TO MAKE SURE THAT THEY WORK!!!

Ex) Solve for x. a) $x - 2 = \sqrt{2x - 4}$ b) $x + 3 = \sqrt{3x + 19}$

Simplifying Rational Expressions

1. Factor the numerator(s) and denominator(s) first!! This will help you to see if anything cancels out
2. Cancel out any factors the numerator and denominator have in COMMON

Ex) Express in simplest form. a) $\frac{3x^2+6x}{x^2+5x+6}$ b) $\frac{2x^2-12x}{x^2-4x-12}$

Finding the Vertex of a Quadratic

** The vertex is a coordinate point (x , y)

1. Use the formula $-\frac{b}{2a}$ to find the x value of the vertex (this is also the axis of symmetry)
2. Plug this x value back into the equation to solve for y → you now have the x and y coordinate of the vertex

Ex) Find the vertex. a) $y = x^2 + 12x + 32$ b) $x^2 + 4x - 12 = 0$

Finding the x-intercept(s) and y-intercept(s) of a Quadratic

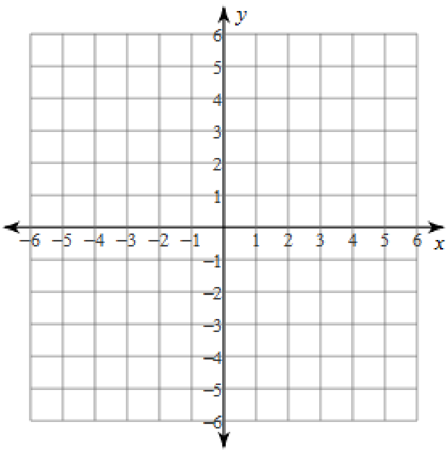
x-intercept(s) ** These are the places that the graph of a quadratic (a parabola) touch the x-axis 1. Set y equal to 0 2. Solve for x	Ex) Find the x-intercepts. a) $y = x^2 - x - 12$ b) $y = x^2 - 2x - 8$
y-intercept(s) ** These are the places that the graph of a quadratic (a parabola) touch the y axis 1. Set x equal to 0 2. Solve for y	Ex) Find the y-intercepts. a) $y = -x^2 - 4x + 2$ b) $y = x^2 - x - 12$

Graphing a Linear Function

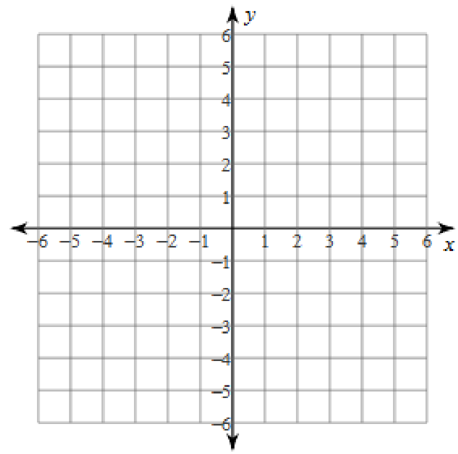
1. Rearrange the equation so that it is in the form $y = mx + b$ if it is not in this form already
2. Plot "b" on the y-axis. This is the y-intercept
3. From there, use the slope to plot additional points ($m = \frac{rise}{run}$)

Ex) Graph each.

a) $x - 2y = -2$



b) $5x + 2y = -6$



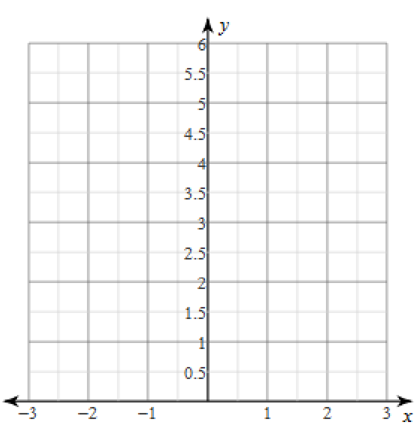
Graphing Quadratics in Standard Form (standard form: $ax^2 + bx + c$ like $x^2 + 3x + 7$ or $2x^2 - 4x + 1$)

1. Find/plot the axis of symmetry using the formula $-\frac{b}{2a}$
2. Plug this x value into the original equation to get y, and these two values make up the vertex (x, y). Plot it.
3. Plot additional points- make a chart of x and y values. Choose two x values that're less than the x value in the
4. Reflect these two points

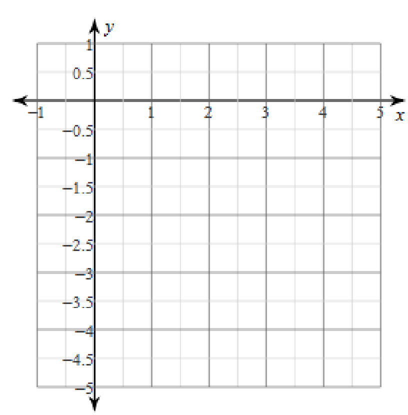
** You can also find the x and y intercepts to use and points for your graph **

Ex) Graph each.

a) $y = x^2 - 2x + 2$



b) $y = x^2 - 2x - 3$



Finding the Solution to Systems of Inequalities by Graphing (not on a number line)

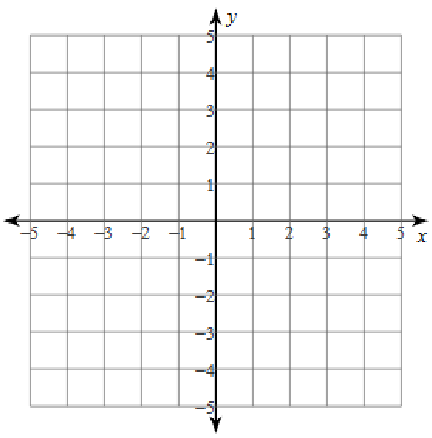
For each:

1. Rearrange the inequality so that y is by itself on one side
2. Graph as if the inequality were an = sign-the line is dashed if the sign is $<$ *or* $>$ and is solid if the inequality is $\leq$ *or* $\geq$
3. Choose a point on the graph that the line does NOT touch and plug the x and y value into the inequality – if your result is a true statement, shade the entire graph on the side of the line INCLUDING the point, if it is a false statement shade the entire side of the graph EXCLUDING the point

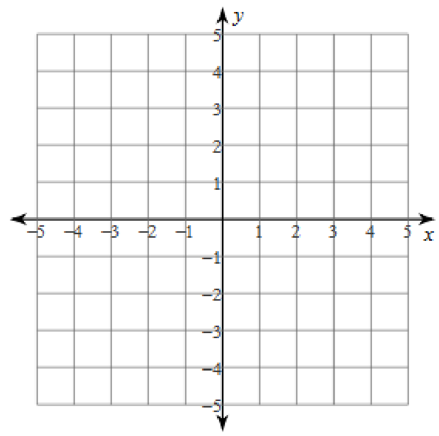
** Your solution is the area that was shaded for all three lines. There is no solution to be written, this itself is the solution.

Ex) Show the solution of the system of inequalities by graphing

a) $y \leq x + 2$ $\quad$ $y > 5x - 2$



b) $y \leq -4x - 1$ $\quad$ $y > -x + 2$

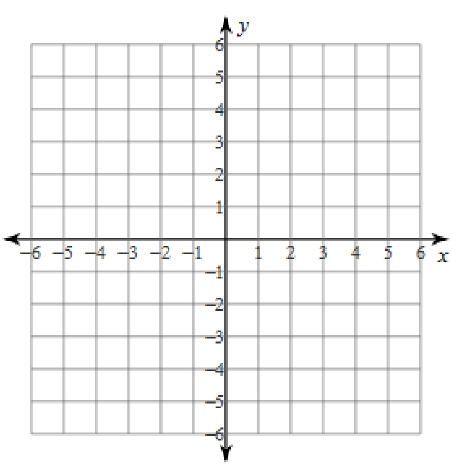


Graphing Absolute Value Equations on a Graph (not on a number line)

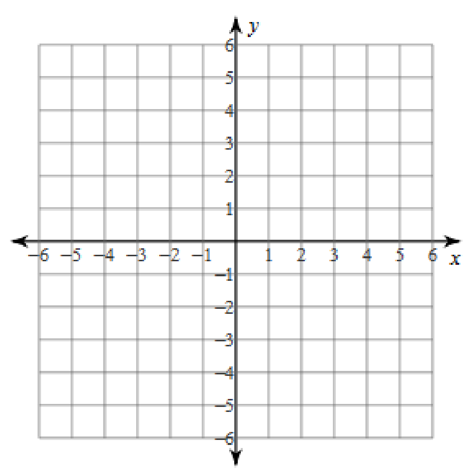
1. Find the vertex of the graph by setting what's inside the absolute value = to zero and solving for x. plug this back into the equation to solve for y to get the (x, y) coordinate that is the vertex
2. Make an x y chart – be sure to pick some x values that make what's inside the absolute value negative!!!
3. Plot the points. The graph should look like a V

Ex) Graph each.

a) $y = |x - 2| + 2$



b) $y = |x - 2| + 1$



Solving and Graphing Inequalities on a Number Line

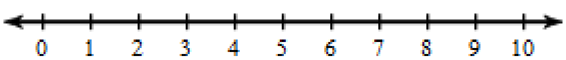
Basic Algebraic Inequalities

1. Solve for x, pretending that the inequality symbol is essentially an equals sign
2. Write appropriate intervals on a number line
3. Draw a small circle above the value that is on the other side of the inequality sign from x
4. Fill in the circle if the inequality is $\leq \quad or \quad \geq$ and leave the circle empty if the inequality is $< \quad or \quad >$
5. Draw an arrow coming from the circle in the direction that the inequality sign is pointing (arrow points to the right if the inequality is $\geq \quad or \quad >$ and points to the left if the inequality is $\leq \quad or \quad <$)

Ex) Solve and Graph the Inequalities

a) $-x - 16 \geq -4x + 2(-3x + 1)$

b) $-6 + 6x < 3(x - 1) - 4$



Inequalities with Absolute Value

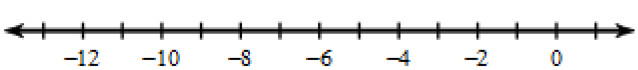
1. Isolate the absolute value part of the inequality onto one side of the inequality sign
2. Place the inequality sign opposite of the one being used onto the other side of the absolute value, so it is surrounded by inequality symbols
3. Next to this new inequality sign, place the same number found on the other side of the inequality, only with the opposite sign
4. Drop the absolute value lines and rearrange things so that ONLY x is in the middle

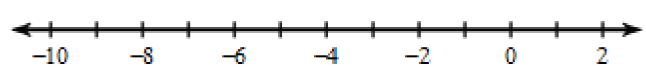
MAKE SURE TO FLIP THE INEQUALITY SIGN IF YOU DIVIDE BY A NEGATIVE

Ex) Solve and graph the given inequalities.

a) $-6|x + 5| + 2 < -16$

b) $4|x + 1| - 1 < 3$





Evaluating Functions

- If you're told to evaluate a function $f(x)$ for, for example, $f(-1)$ → plug -1 in for every x in the function, and solve (for y)
- If you are told to evaluate a function $f(x)$, for example, when $f(x) = 3$ → set the function equal to 3 and solve (for x)

Ex a) Evaluate $f(-\frac{3}{4})$ if $f(x) = 7 - 4x$

Ex b) Evaluate $h(-\frac{1}{2})$ if $h(x) = 12 + 6x$

Ex c) Evaluate $g(x) = 11$ if $g(x) = 2x - 9$

Ex d) Evaluate $k(x) = 18$ if $k(x) = 7x - 3$

Factoring Polynomials

Basic Factoring of Quadratics in the form $\boldsymbol{ax^2 + bx + c}$

If the leading coefficient (a) is 1:

1. Find two numbers that can be multiplied to get c and can be added to get b
2. Put these two numbers in the blanks, using the appropriate sign (x _____)(x ______)

Ex) Factor the expressions. a) $x^2 - 9x + 14$ b) $x^2 - 4x - 32$

If the leading coefficient (a) is greater than 1:

72	
1	72
2	36
3	24
4	18
6	12
$\boxed{8 \quad 9}$	

$$6x^2 + 17x + 12$$
$$6x^2 + 8x + 9x + 12$$
$$(6x^2 + 8x) + (9x + 12)$$
$$2x(3x + 4) + 3(3x + 4)$$
$$(3x + 4)(2x + 3)$$

Steps:

1) Find factors of 72 that add up to 17
2) Rewrite the polynomial so that the middle term is a sum of the 2 factors you found
3) Factor by grouping

Ex) Factor the expressions.

a) $3x^2 + 4x + 1$

b) $3x^2 - 4x - 4$

Difference of Squares

1. Rewrite each term as "something squared" if it isn't written that way already
2. Factor using parentheses with one of each term that is being squared included. One set of ( ) will have a + sign and one set will have a – sign

Ex) Factor the expressions. a) $16x^2 - 36$ b) $4x^2 - 64$

When there is a common term

1. Take out all common factors of the numbers and/or the variables and write it outside of a set of parentheses
2. Inside of the parentheses, write what remains of the expression once the common term(s) is taken out
3. ALWAYS CHECK to see if that you have inside of the parentheses can be factored farther

Ex) Factor the expressions. a) $8x^3 - 24x^2 - 32x$ b) $4x^3 - 14x^2 - 20x$

Factoring by Grouping ** This will work if the polynomial has 4 parts

$$3x^3+2x^2-3x-2=0$$
$$\left(3x^3+2x^2\right)-(3x+2)=0$$
$$x^2(3x+2)-(3x+2)=0$$
$$\left(x^2-1\right)(3x+2)=0$$
$$(x+1)(x-1)(3x+2)=0$$

Ex) Factor the expressions by grouping.

a) $x^3 + 2x^2 - 4x - 8$ b) $x^3 - 5x^2 - 4x + 20$

Solving Quadratics (or polynomials) that can be Factored

1. Factor the polynomial using one of the methods above
2. Set each factored piece equal to zero and solve for x

Ex) Solve for x. a) $x^2 - 9x + 14 = 0$ b) $4x^2 - 64 = 0$

Solving Quadratics that cannot be factored – Need to Use the Quadratic Formula

Quadratic Formula: $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

**Helpful to write what a, b and c equal right off of the bat to help you plug values into the quadratic formula

**The quadratic formula will ALWAYS work for solving quadratics

Ex) Solve for x using the quadratic formula.

a) $x^2+2x-1=0$

b) $x^2-x-6=0$

Set Builder Notation

Universal Set U	the set of all of the elements under consideration
Union $\cup$	The union of two sets A and B are all of the elements included in set A or in set B (not JUST the elements A and B have in common!!!)
Intersection $\cap$	The intersection of two sets A and B is the set of all of the elements in both A and B (Only the elements that both sets share!!)
Integers Z	Positive and negative whole numbers, not including decimals or fractions that cannot be written as a whole number.
Whole Numbers W	All integers from 0 to ∞

The easiest way to organize your information is to:

1. Write out all of the elements that are included to "decode" the info given in set builder notation
2. make a Venn diagram to organize the information

Ex) Let $U=\{x\,|\,x\in Z,-5\le x<7\}$

$A=\{x\,|\,x\in Z,-5\le x\le 1\}$

$B=\{x\,|\,x\in Z,-2<x\le 5\}$

$C=\{x\,|\,x\in W,x\le 4\}$

Find: a) $A\cup B$

b) $A\cap C$

c) $(A\cap B)\cap C$

Solving Systems of Equations

Solving Systems of Equations using Elimination

Steps:

1. Place both equations in Standard Form, Ax + By = C.
2. Determine which variable to eliminate with Addition or Subtraction.
3. Solve for the remaining variable.
4. Go back and use the variable found in step 3 to find the second variable.
5. Check the solution in both equations of the system.

Solving Systems of Equations using *Substitution*

Steps:

1. Solve <u>one</u> equation for <u>one</u> variable (y= ; x= ; a=)
2. Substitute the expression from step one into the other equation.
3. Simplify and solve the equation.
4. Substitute back into either original equation to find the value of the other variable.
5. Check the solution in both equations of the system.

Solving Systems of Equations by Graphing:

1. Graph each line
2. Your solution will be the point where the lines *intersect*

Ex) Find the solution to the systems of equations.

a) $-6x - y = 10$
$-3x - 2y = 2$

b) $9x + 2y = -15$
$-3x - 3y = -9$

c) $3x - 3y = 3$
$y = 11 - 2x$

Calculating Percentages from Word Problems

**Be sure to take into account the # of occurrences you are being asked to give the % of as well as the total # of occurrences.

Ex a) A baseball team won 80 games and lost 20. What percent of the total games did they win?

Ex b) A student kept 10 pieces of candy and gave away 40 pieces. What percent of the total amount of candy did the student keep?

Simplifying Radical Expressions

Rules:

1. If a radical is not a perfect square (like $\sqrt{9}$), try to find 2 numbers you can multiply to get the number inside of the radical, one of which has to be a perfect square. Use this information to <u>rewrite/simplify the radical</u>
2. When <u>adding/subtracting radicals</u> – can only do this if the radicals are THE SAME. The number inside of the radical does not change, you only add/subtract the coefficients (think like terms)
3. When <u>multiplying radicals</u>- you are allowed to multiply the numbers inside of a radical to get a new radical
4. If there is a <u>radical in the denominator:</u>
   a) If the only thing in the denominator is a radical, multiply the whole fraction by that radical over itself
   b) If the denominator is an expression involving a radical, multiply the whole fraction by that expression over itself, but with the OPPOSITE sign that the expression involves in the denominator

Ex) Simplify. a) $\frac{3}{7+\sqrt{2}}$ b) $\sqrt{8}(2\sqrt{6}+3\sqrt{5})$ c) $\sqrt{36}+3\sqrt{12}-4\sqrt{3}$

Finding the Slope of a Line Given Points

Plug the points into the formula $\frac{y_1-y_2}{x_1-x_2}$

Ex) Find the slope of the line that contains the points given.

a) $(17,7)$ *and* $(18,-12)$

b) $(-20,2)$ *and* $(-2,4)$